Exploring QCD phase diagram at vanishing baryon density on the lattice

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RIKEN lunch seminar

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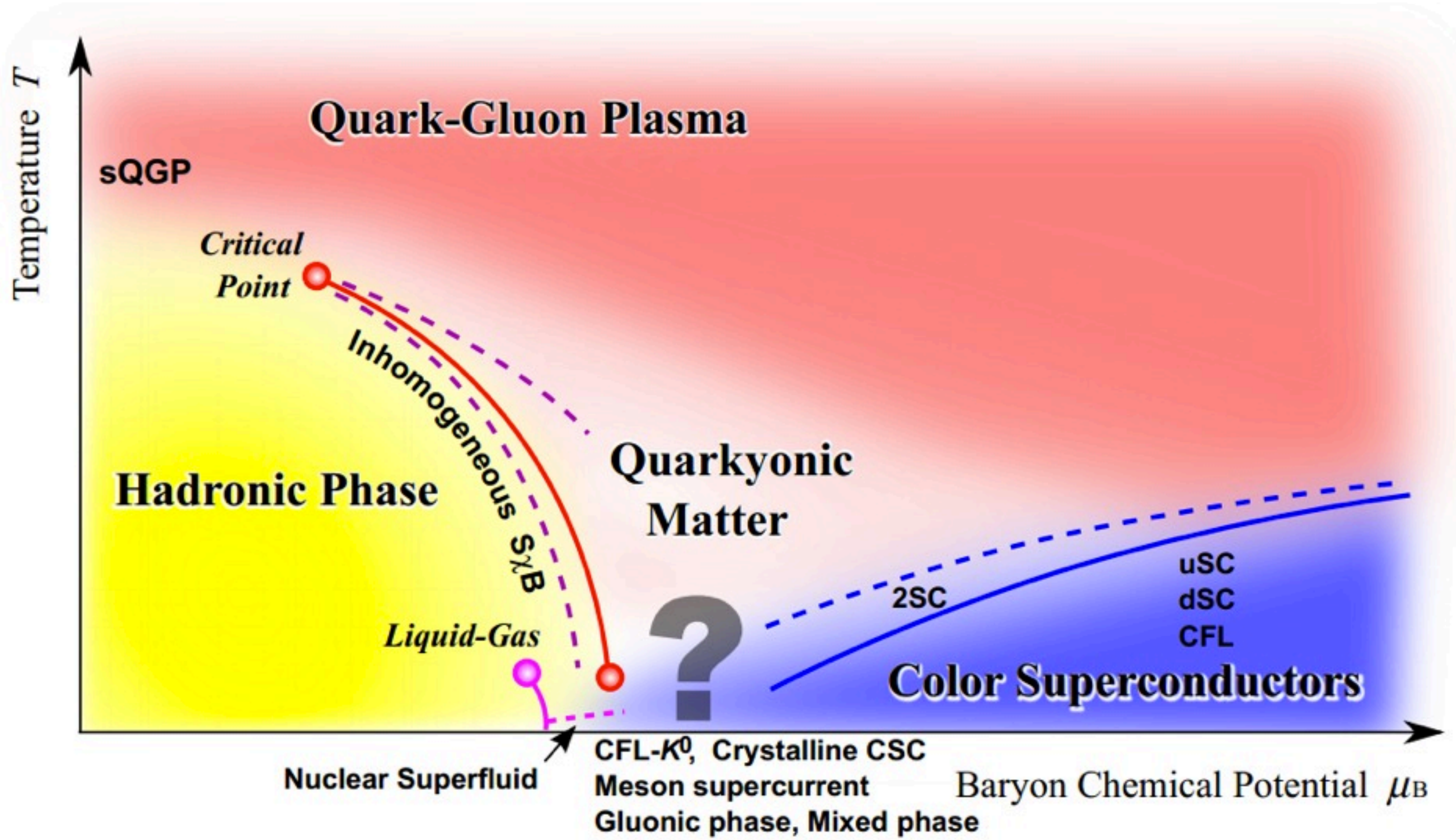
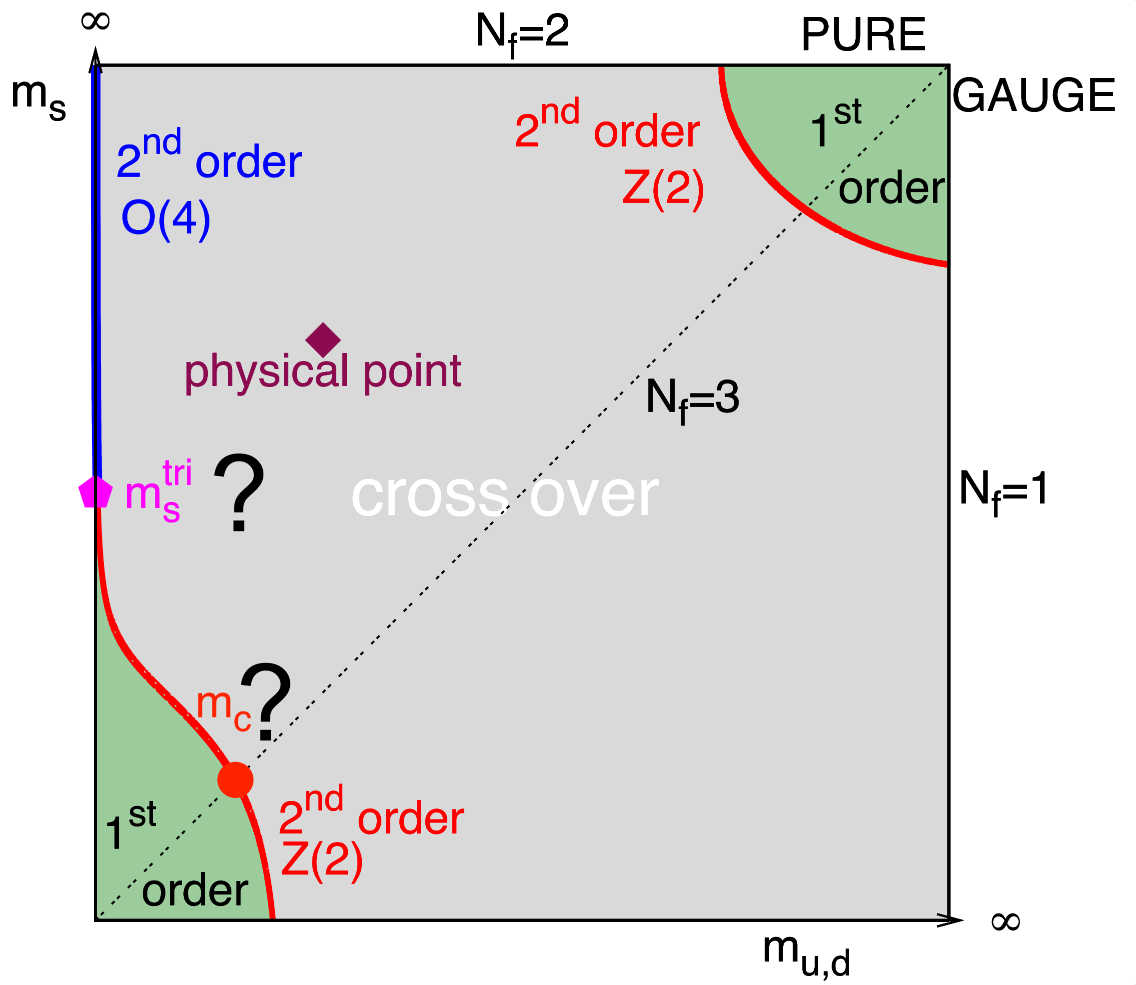
sketched QCD phase diagram



Fukushima & Hatsuda '10

QCD phase diagram at mu=0

columbia plot:



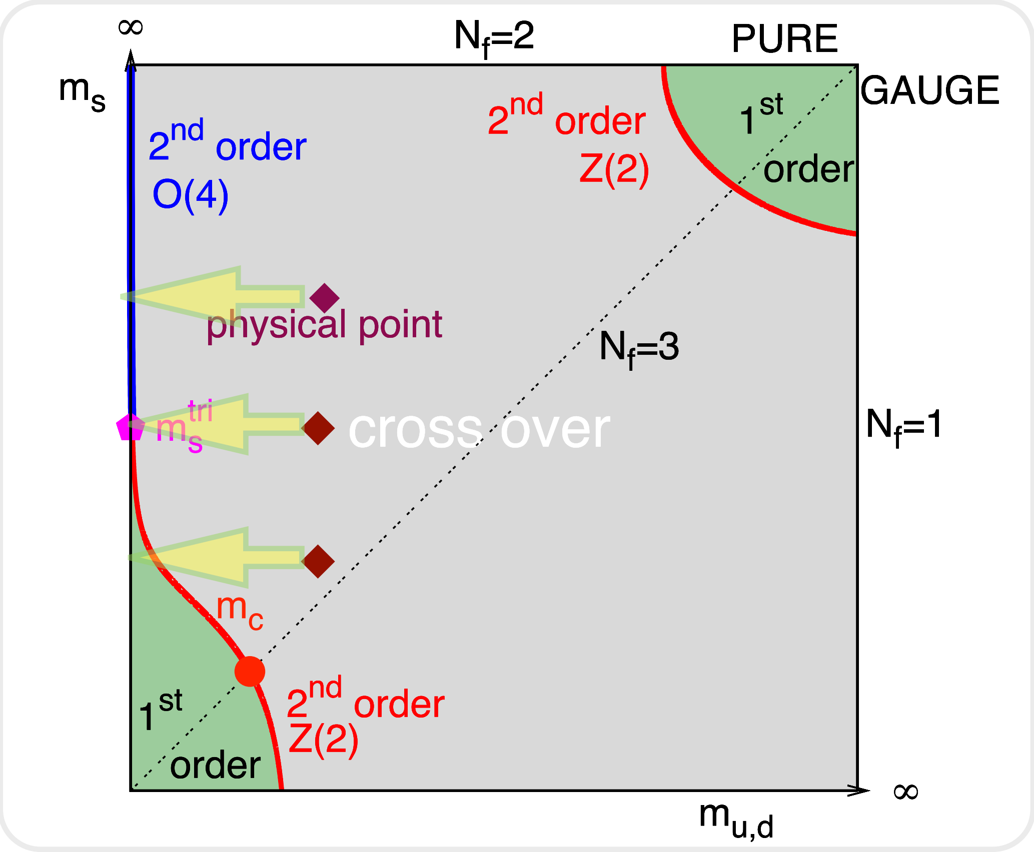
- N_f=2+1 theory: at m=0 or ∞ has a first order phase transition
- Intermediate quark mass region an analytic cross over is expected
- At physical quark masses, a cross over is confirmed
- Critical lines of second order transition
  N_f=2: O(4) universality class
  N_f=3: Ising universality class

★ The fundamental scale of QCD: chiral phase transition T_c ?

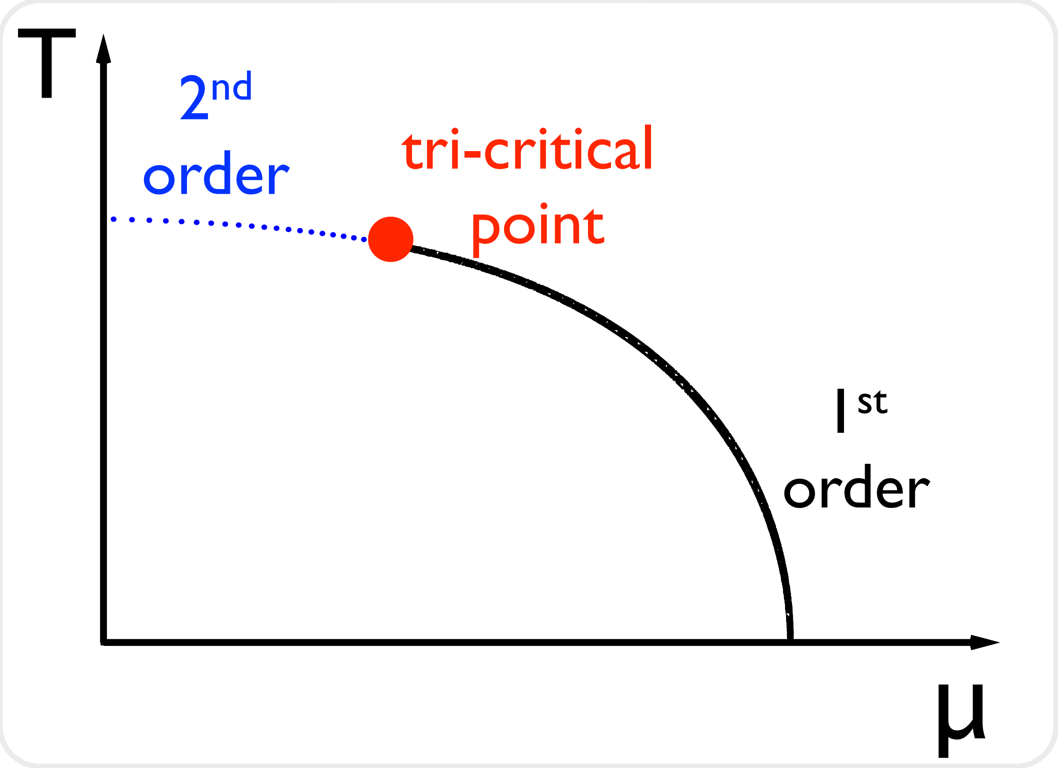
★ $m_s^{tri} < m_s^{phy}$ or $m_s^{tri} = m_s^{phy}$ or $m_s^{tri} > m_s^{phy}$?

The proximity of 2nd order Z(2) line to the physical point ?

QCD phase transition in the chiral limit (m_l=0)



$m_s^{phy} > m_s^{tri}$



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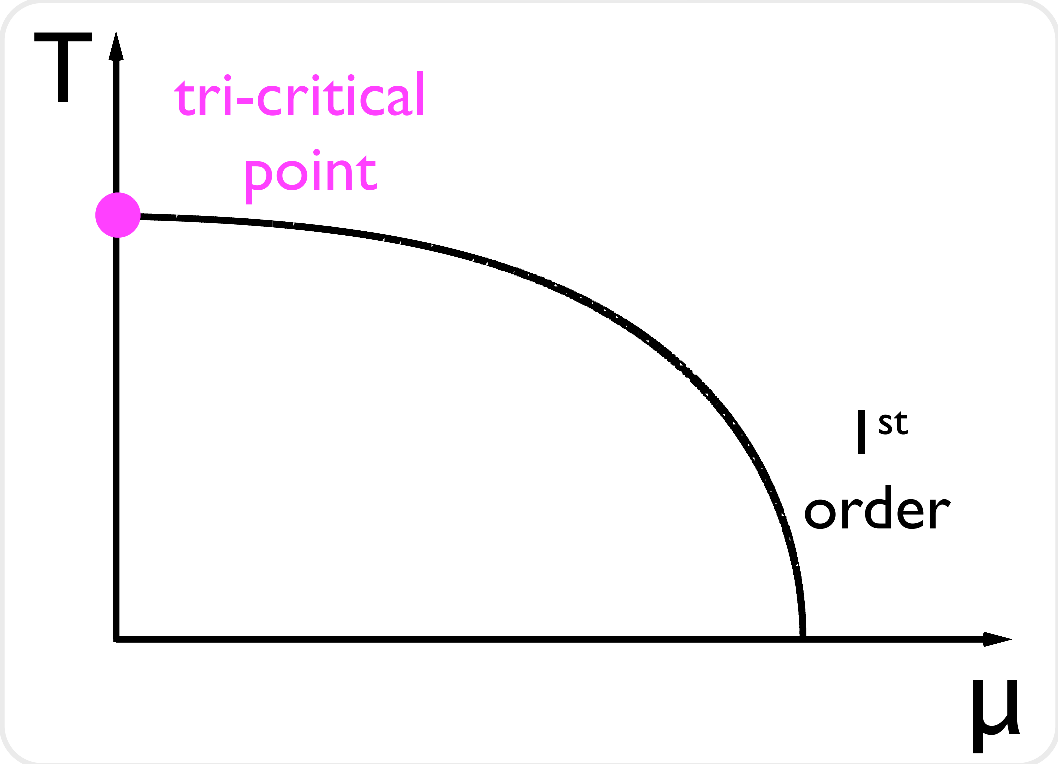

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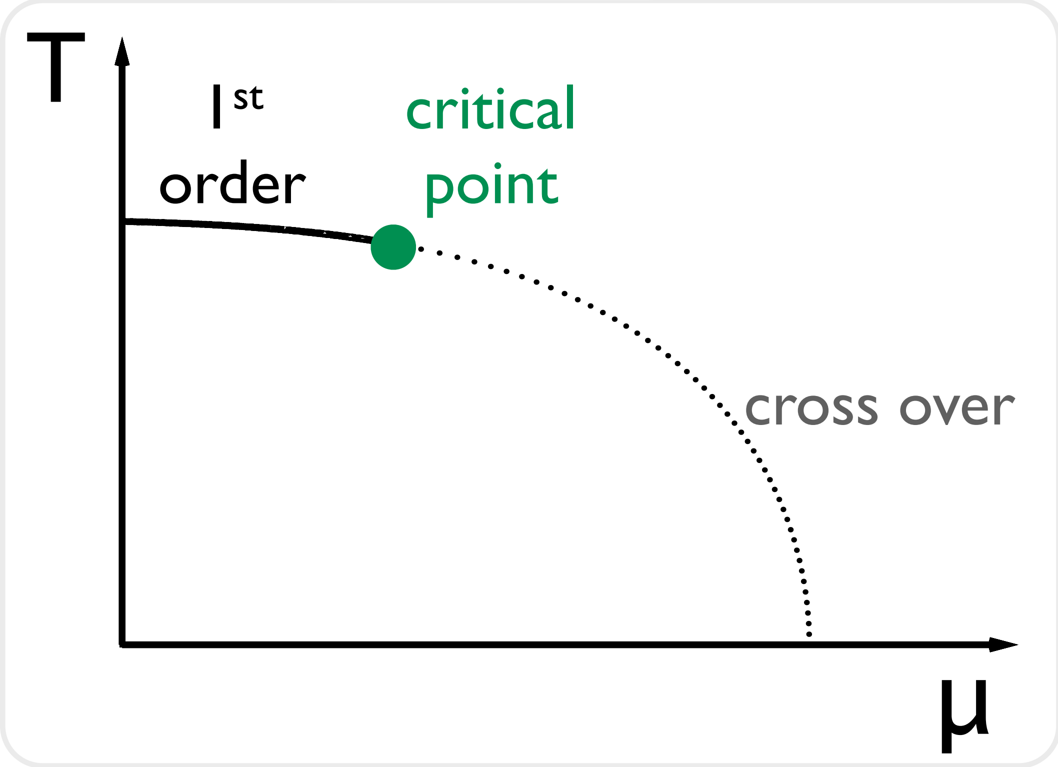
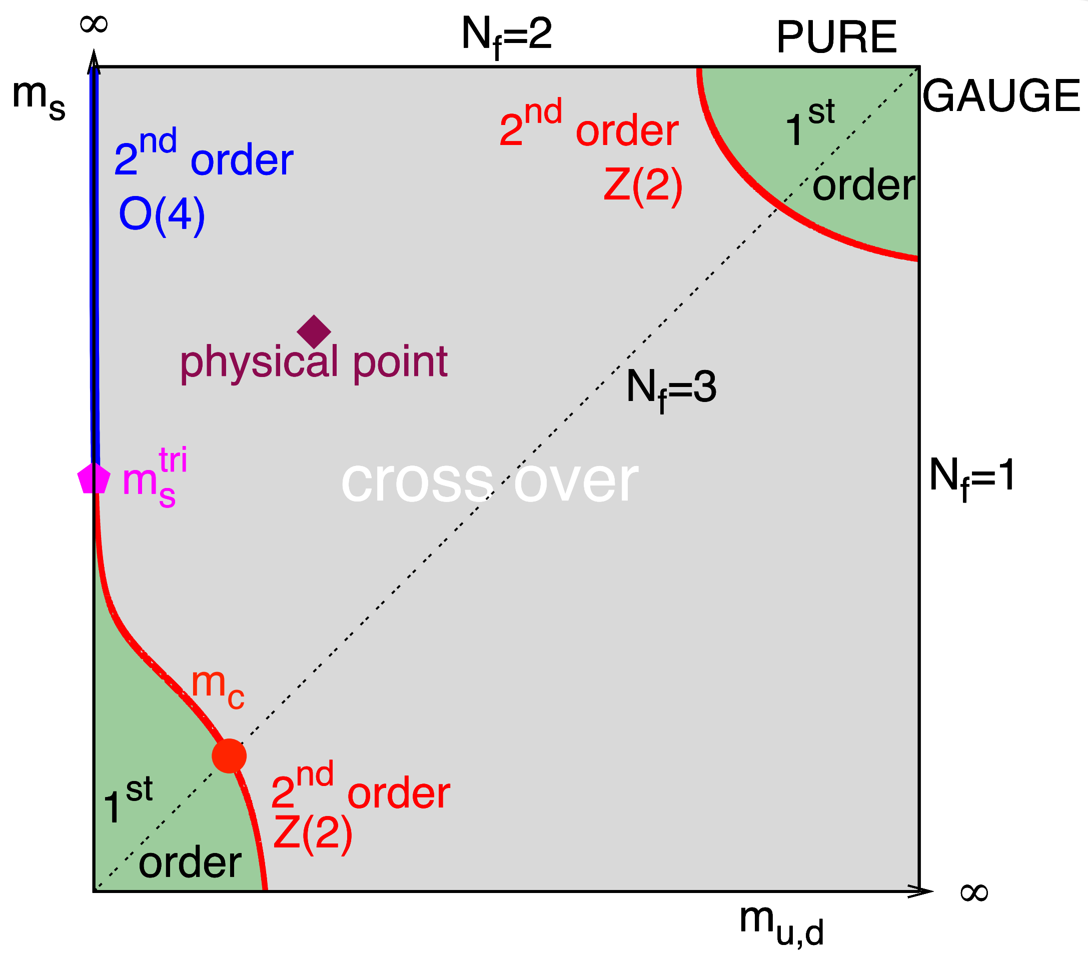
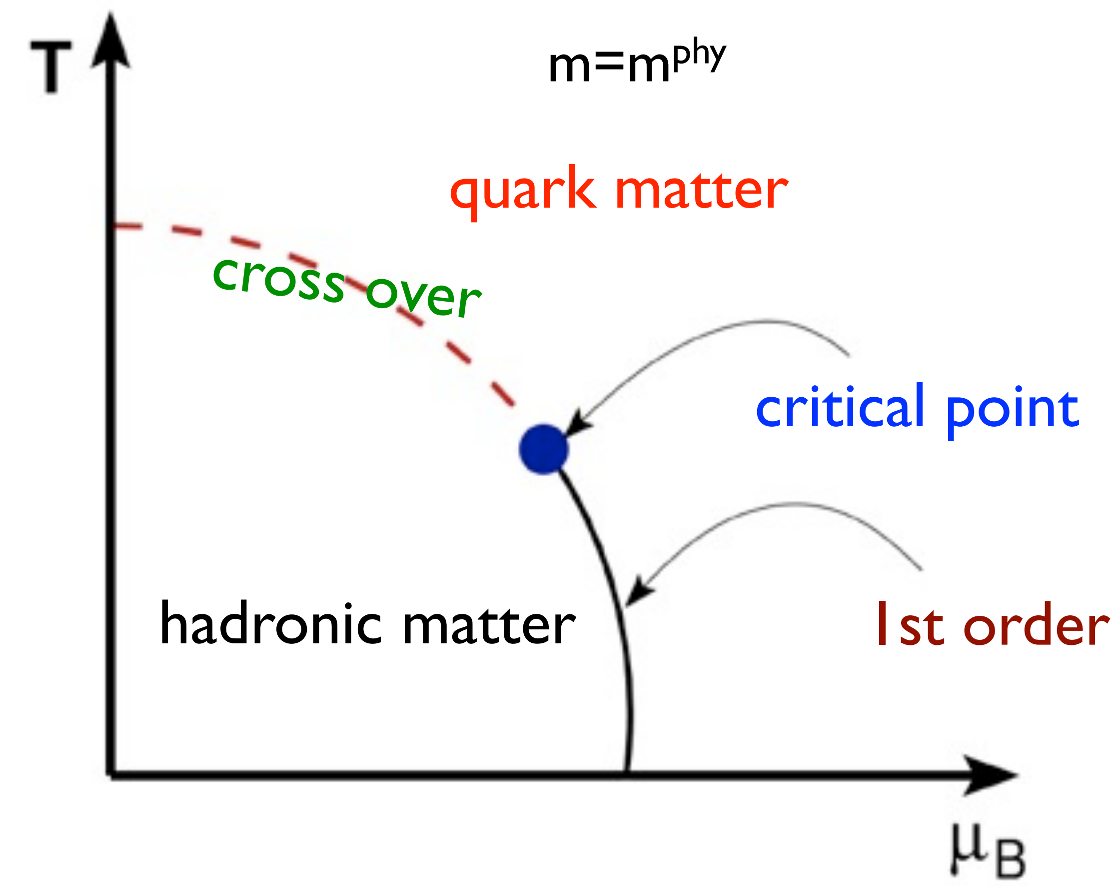
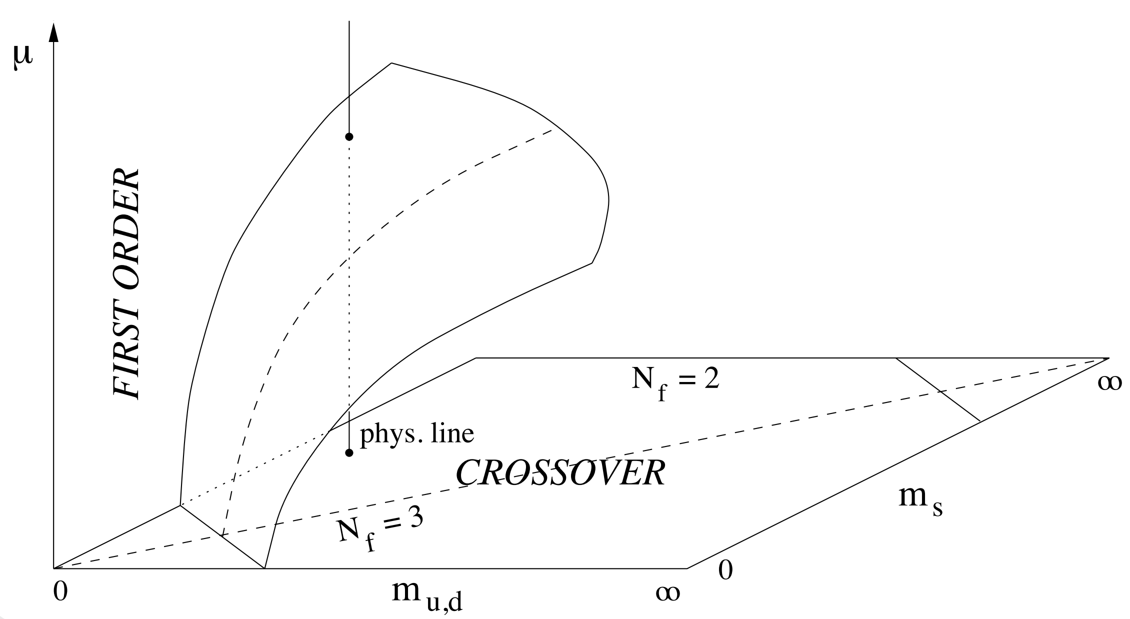


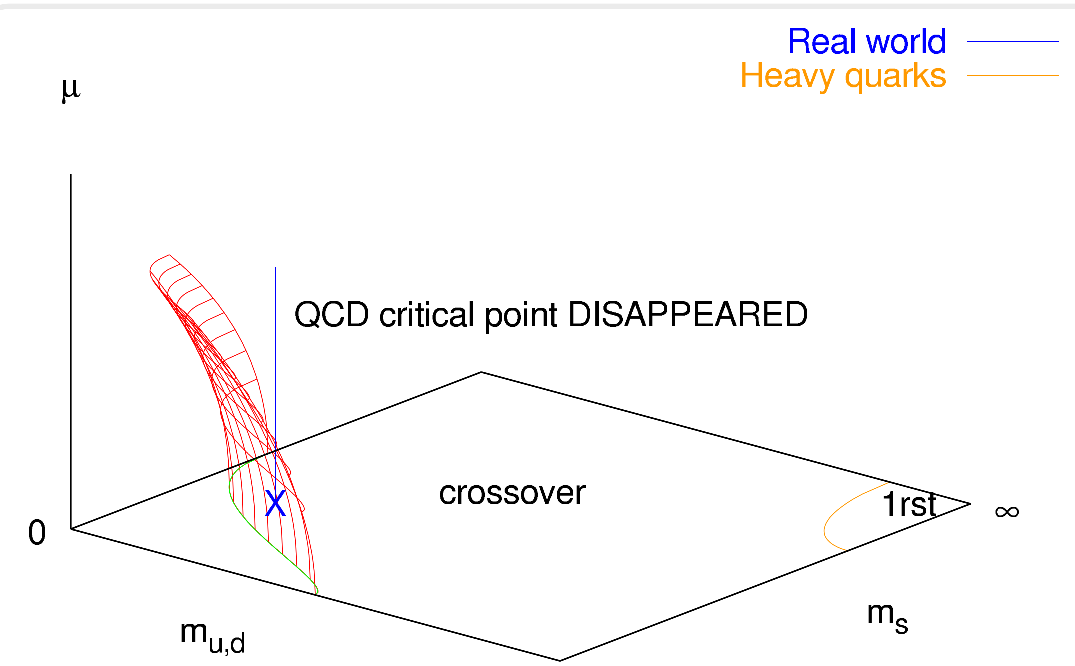
QCD phase transition at the physical point





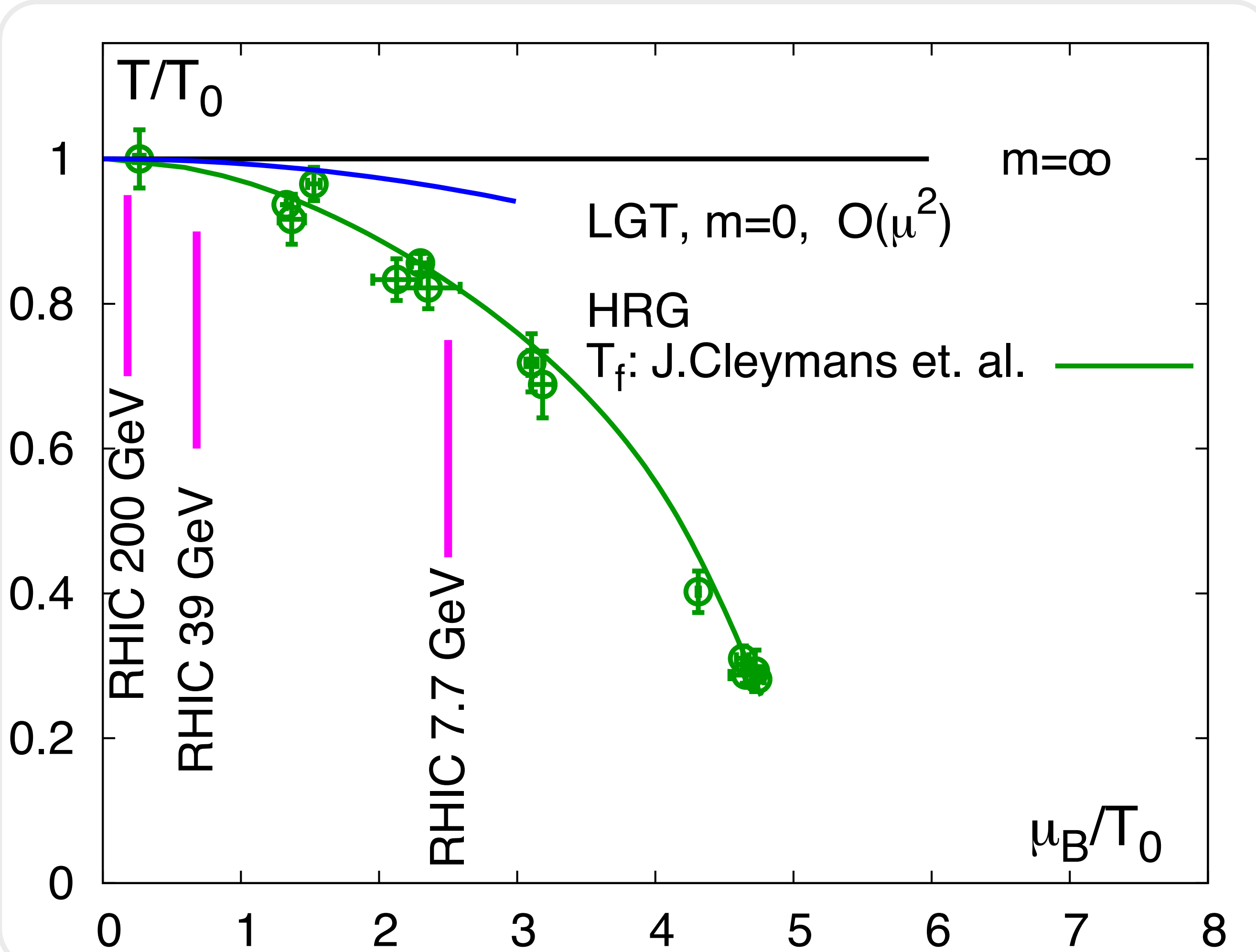


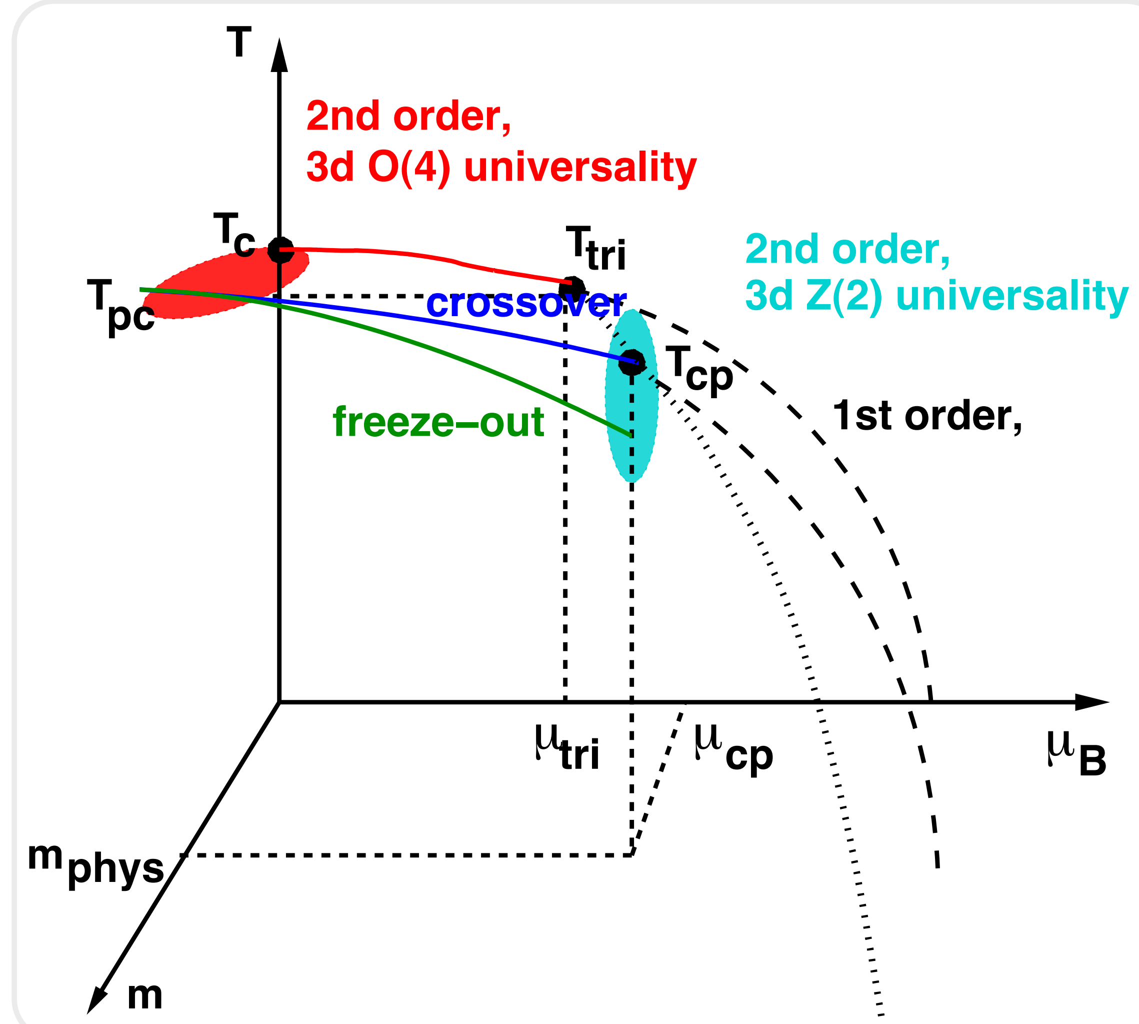
Karsch et al., '03



de Forcrand & Philipsen, '07

feasibility to discover critical point experimentally

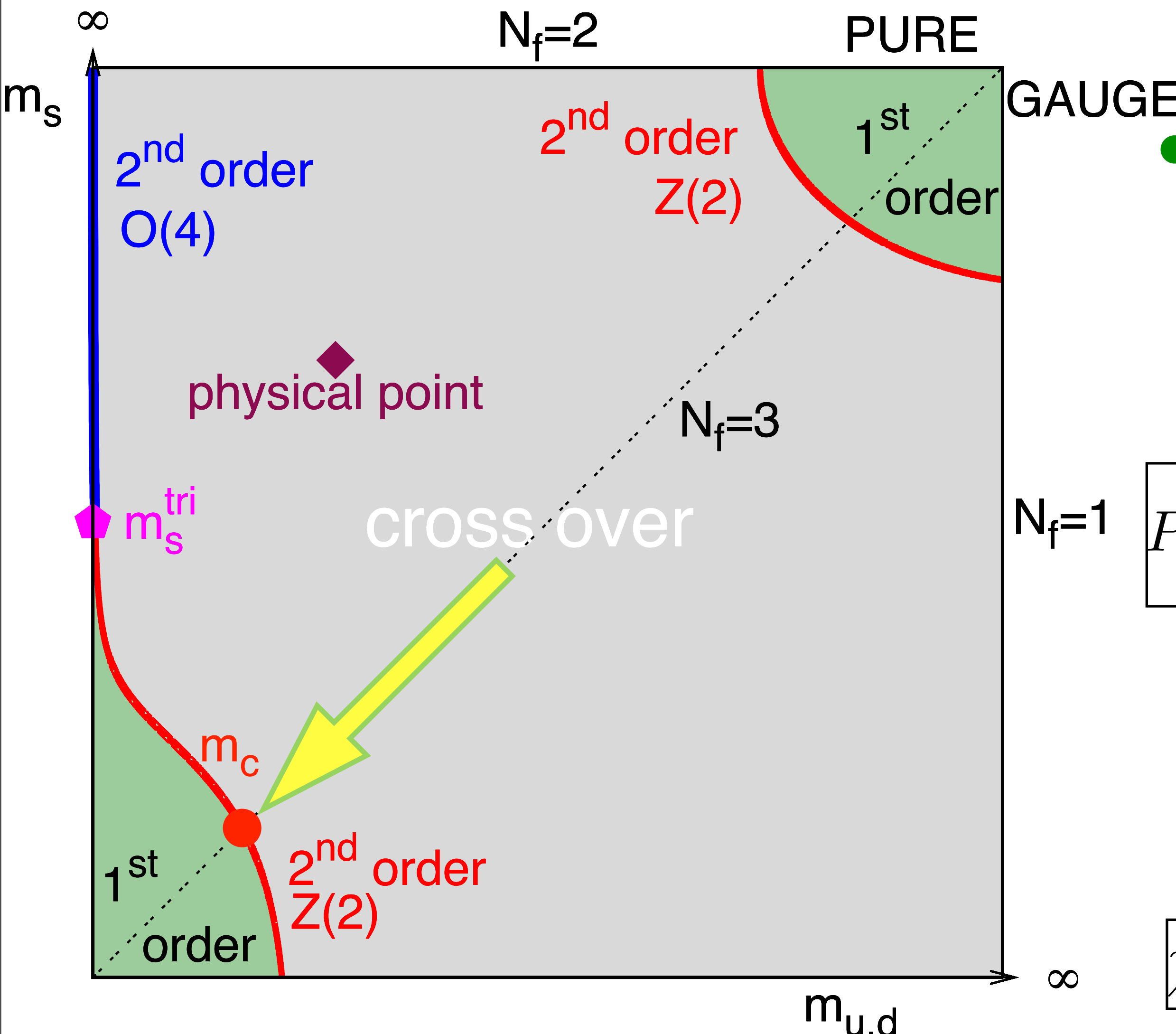




F. Karsch CPOD '11

- Ongoing Beam Energy Scan program at RHIC to discover the critical point (CP)
- Chiral phase transition line from universal scaling analysis on lattice QCD
- Only if the chiral phase transition line is close enough to the freeze out line, there is a hope that the CP can be discovered through heavy ion collision experiments

approaching chiral limit in Nf=3 QCD



- 1st order phase transition

distribution of observable X

$$P(x) \propto \left(\exp\left(-\frac{(x-X_+)^2}{2c/V}\right) + \exp\left(-\frac{(x-X_-)^2}{2c/V}\right)\right)$$

susceptibilities of observable X

$$\chi_X = V\left(\langle x^2\rangle\right) - \langle x\rangle^2\right) = V(X_+ - X_-)^2 + c$$

- 2nd order phase transition belongs to Z(2) universality class

$$M= (\langle \bar{\Psi}\Psi\rangle + r\,\Box)\Big|_{T=Tc,mc} \sim (m-m_c)^{1/\delta}$$

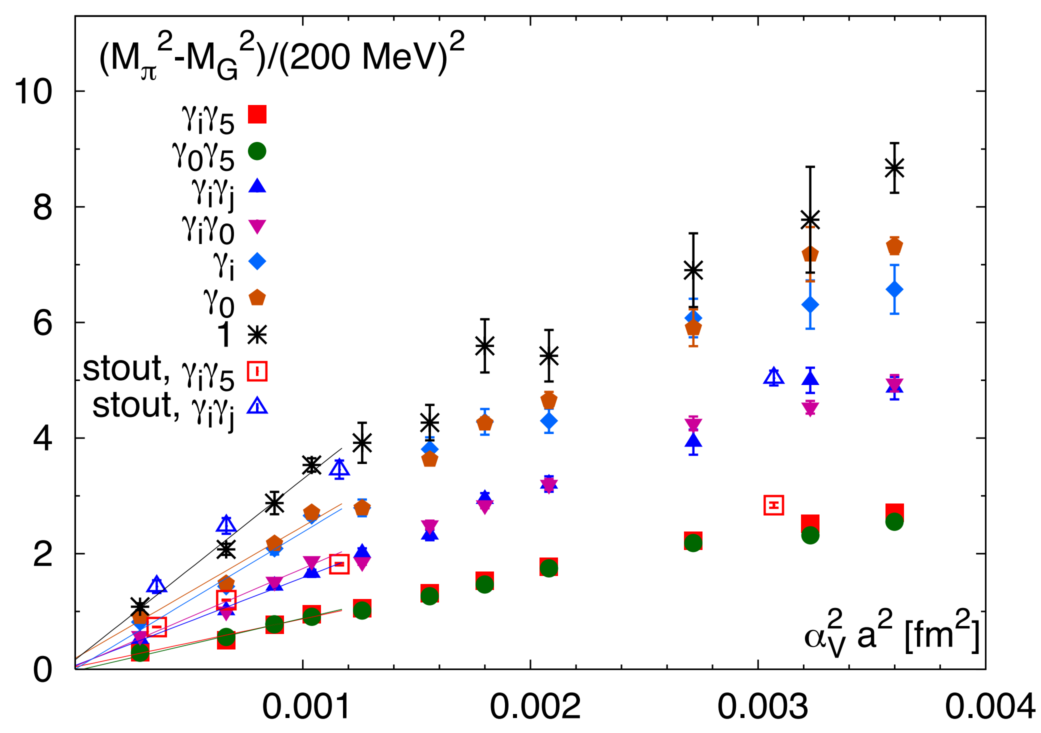
$$\chi_q/T^2\Big|_{T=Tc,mc} \sim (m-m_c)^{1/\delta-1}$$

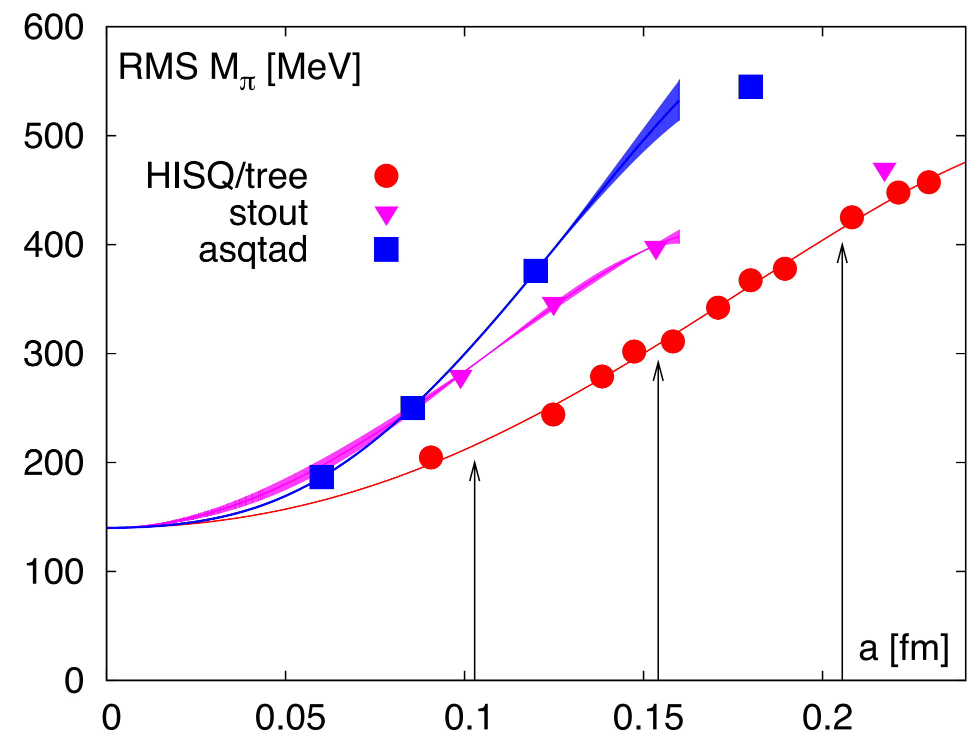
the value of m_π investigated so far...

Dependence on the choice of action

- Naive action: $N_\tau = 4 \Rightarrow m_\pi^c \approx 290$ MeV F. Karsch et al., Nucl.Phys.Proc.Suppl. 129 (2004) 614
- Naive action: $N_\tau = 6 \Rightarrow m_\pi^c \approx 140$ MeV *P. de Forcrand et al, PoS LATTICE2007 (2007) 178*
- p4fat3 action: $N_\tau = 4 \Rightarrow m_\pi^c \approx 67$ MeV F. Karsch et al., Nucl.Phys.Proc.Suppl. 129 (2004) 614
- stout action: $N_\tau = 6 \Rightarrow m_\pi^c \lesssim 50$ MeV G. Endrodi et al., PoS LAT2007 (2007) 228

Highly Improved staggered quarks (HISQ) used





HotQCD, PRD85(2012)054503

Lattice setup

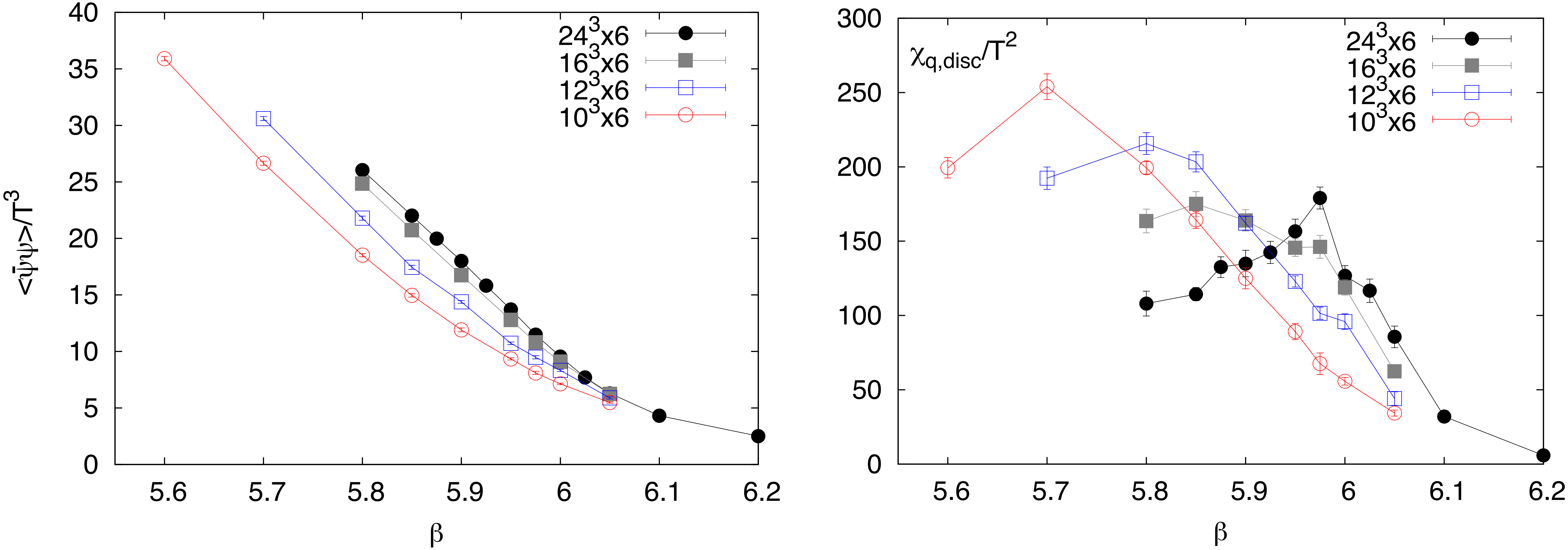
★ Highly Improved staggered fermions/tree action used

★ 3 degenerate quarks, m_π down to 80 MeV

★ N_T=6 lattices

	lattice dim	quark mass	m_π	#T	statistics
	16^3x6	ma=0.0075	230 MeV	17	~8000
Volume dep.	10^3x6	ma=0.00375	160 MeV	9	~12000
	12^3x6	ma=0.00375	160 MeV	8	~12000
	16^3x6	ma=0.00375	160 MeV	7	~12000
	24^3x6	ma=0.00375	160 MeV	12	~8000
	24^3x6	ma=0.0025	130 MeV	5	~8000
	24^3x6	ma=0.001875	110 MeV	7	~8000
	24^3x6	ma=0.00125	90 MeV	7	~8000
Volume dep.	24^3x6	ma=0.0009375	80 MeV	8	~6000
	16^3x6	ma=0.0009375	80 MeV	6	~7000

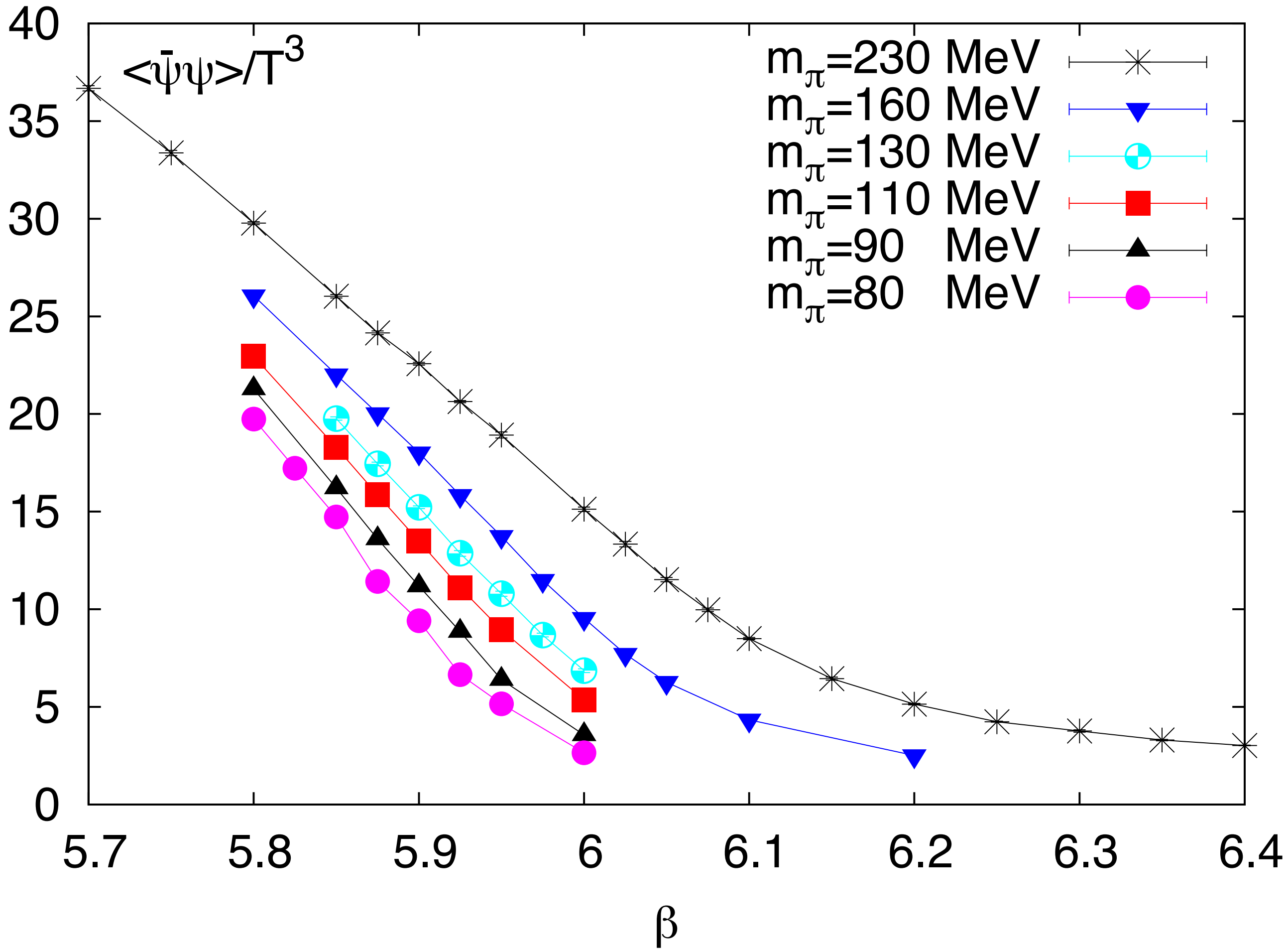
volume dependence with m_π=160 MeV

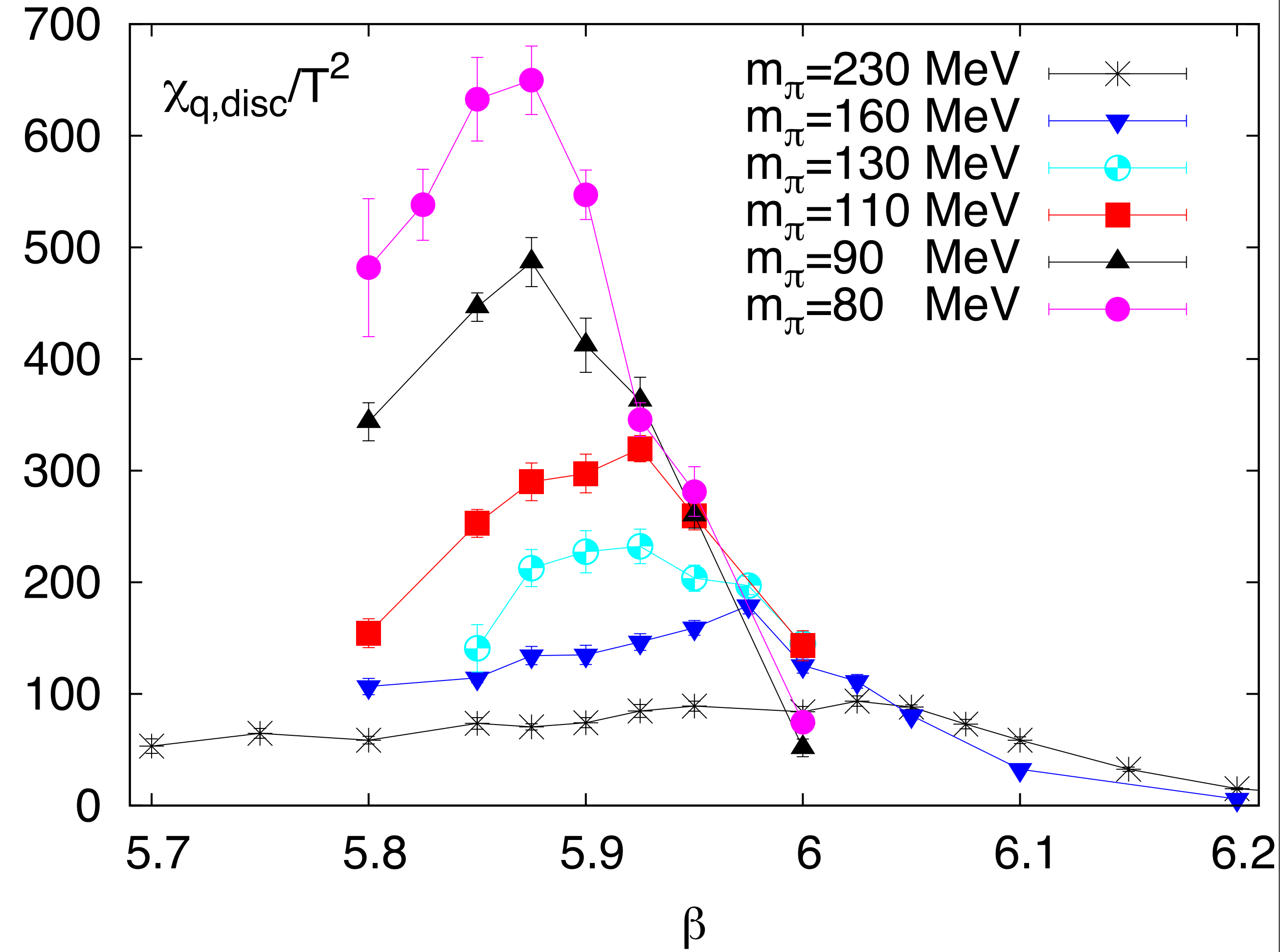


- volume dependence is smaller at higher temperatures

- peak locations in chiral susceptibilities shift to lower temperatures at smaller volume

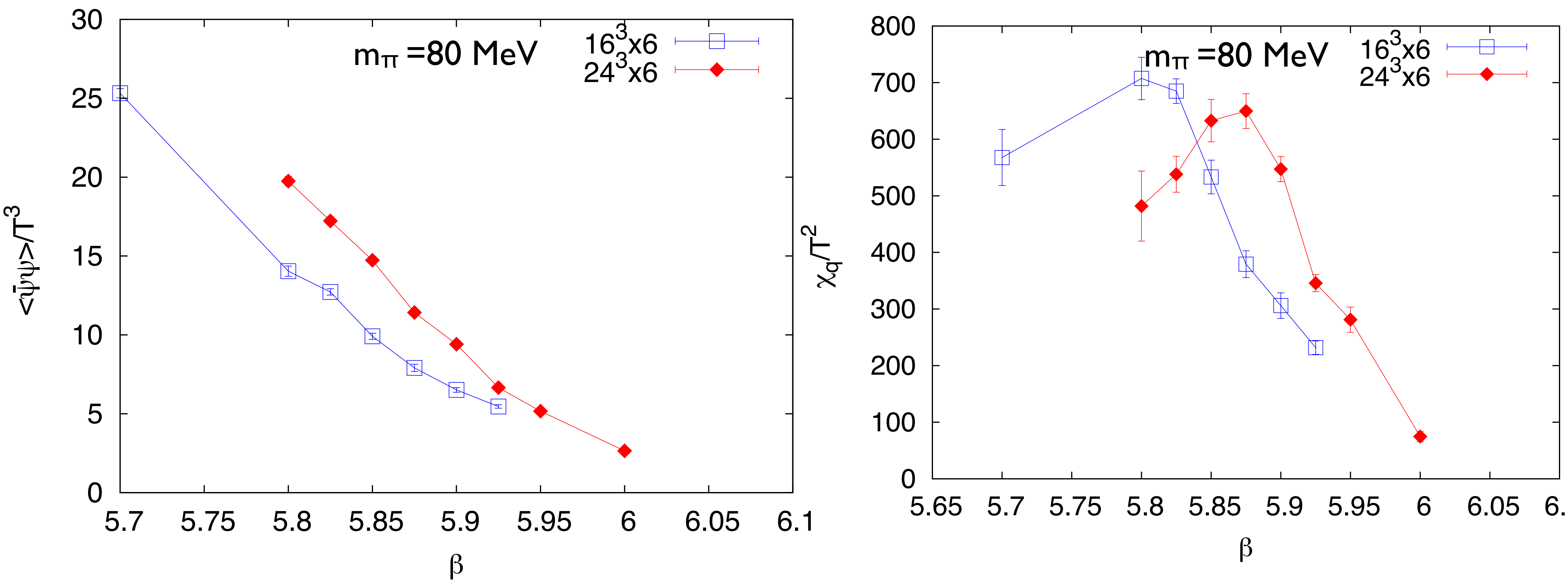
chiral condensates & susceptibilities





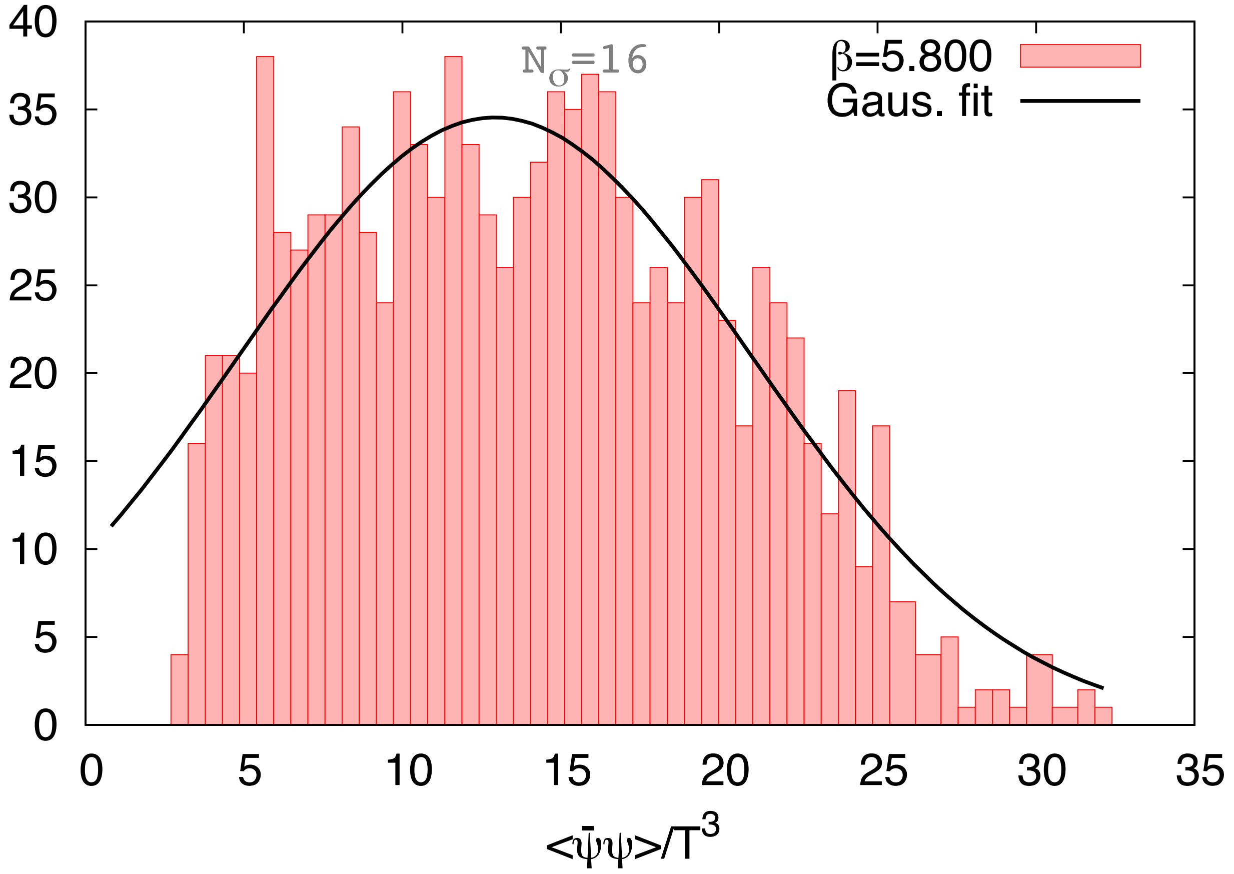
- chiral condensate decreases with temperature
- chiral susceptibility increases at smaller masses

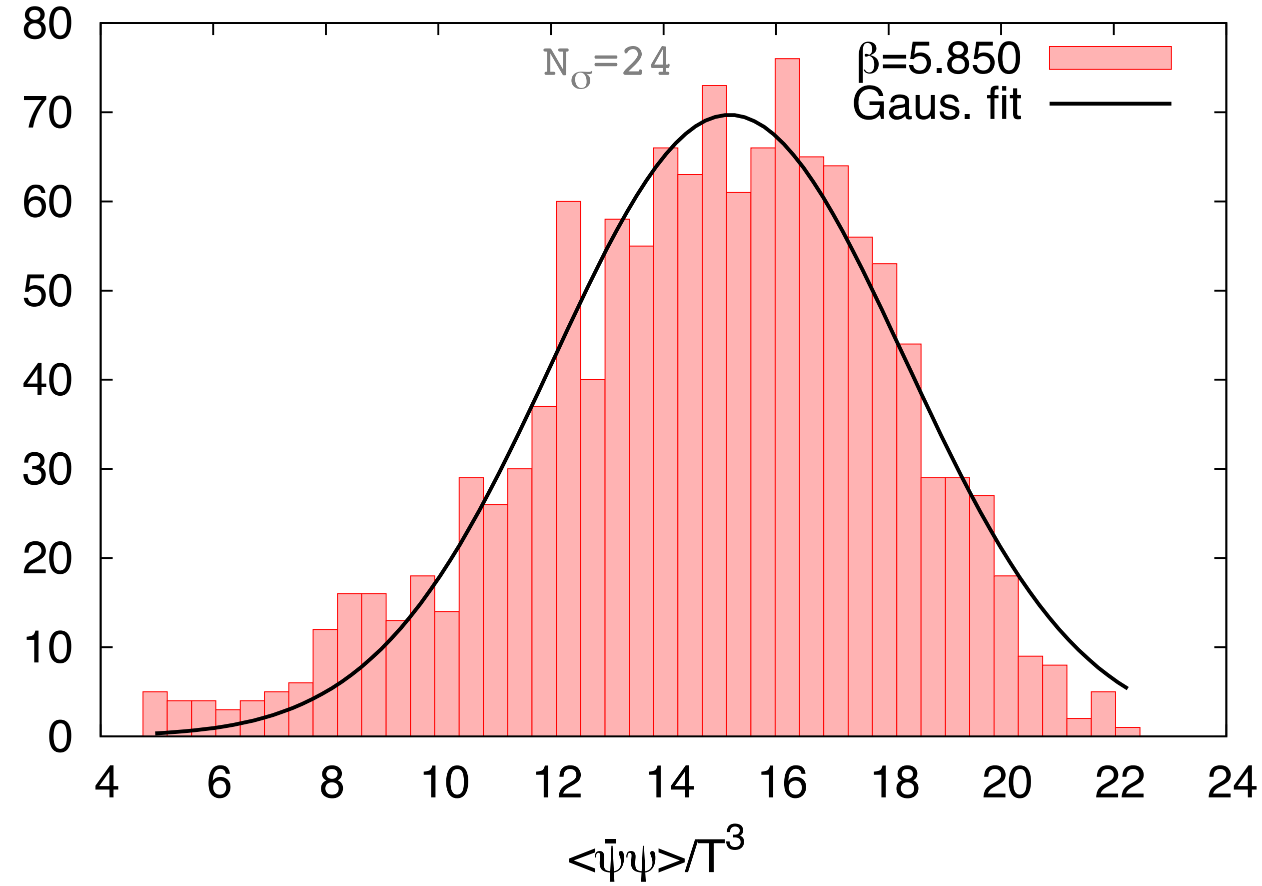
chiral condensates & susceptibilities



- finite volume effects bring chiral condensates down
- No evidence of volume scaling observed from chiral susceptibility

time history of chiral condensates





- No double peak structure is seen

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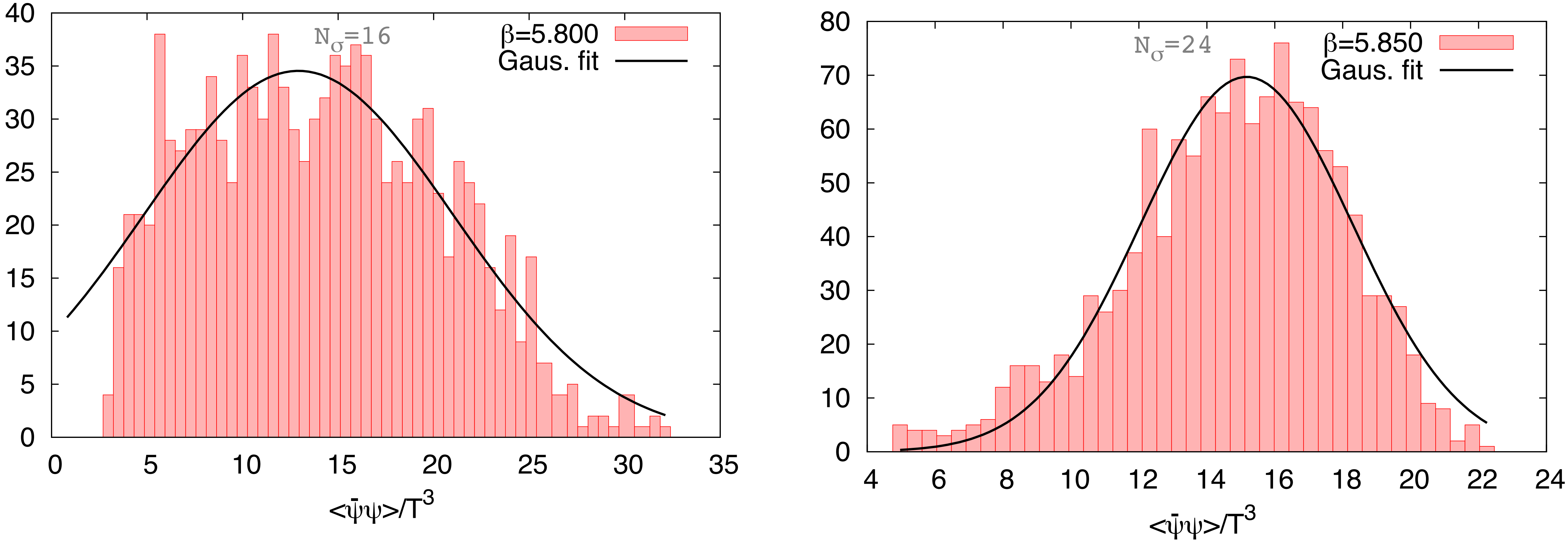
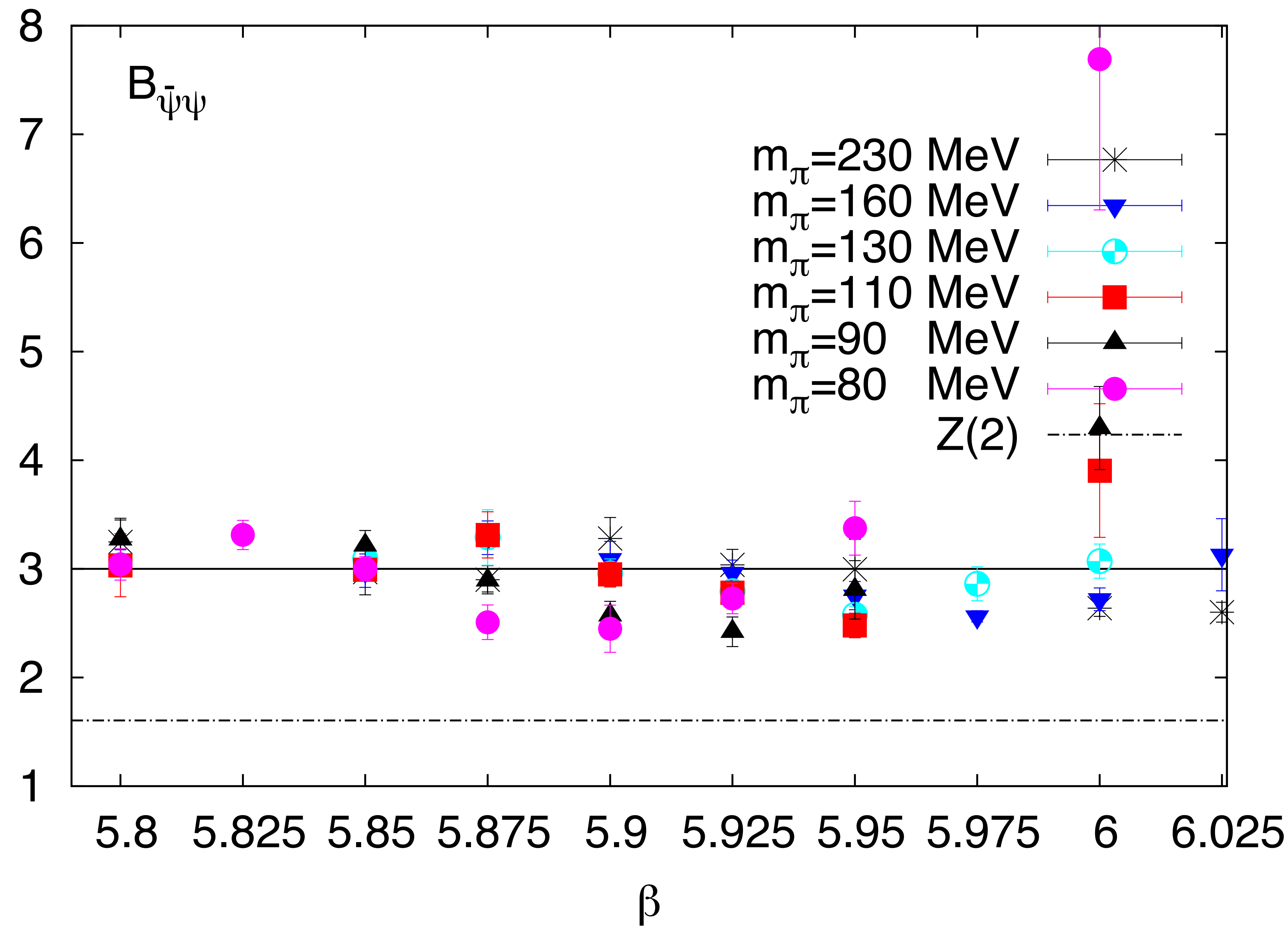


- No double peak structure is seen

- With 230 ≥ m_π ≥ 80 MeV, no direct signal of first order phase transition is found

Binder cumulant of chiral condensate

$$B_{\bar{\psi}\psi} \equiv \frac{\langle(\delta\bar{\psi}\psi)^4\rangle}{\langle(\delta\bar{\psi}\psi)^2\rangle^2}$$

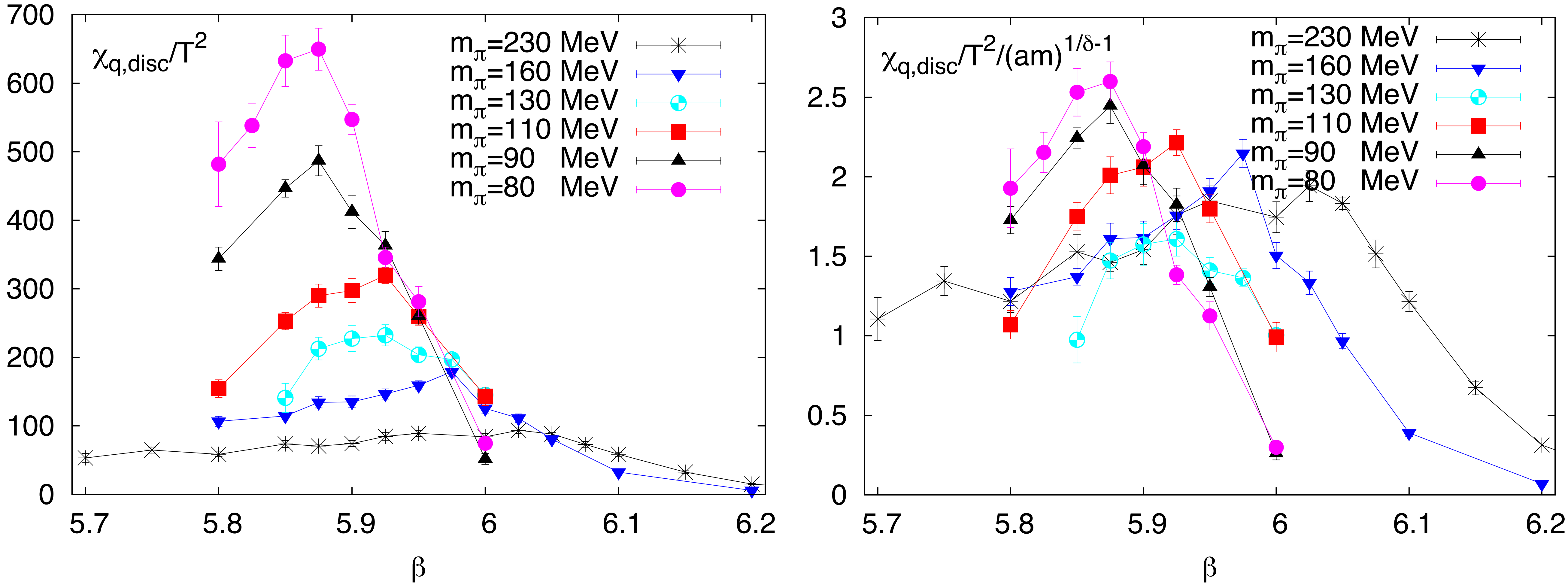


In the crossover region B=3

2nd order transition in the Ising universal class B=1.604

1st order transition B=1

chiral susceptibilities



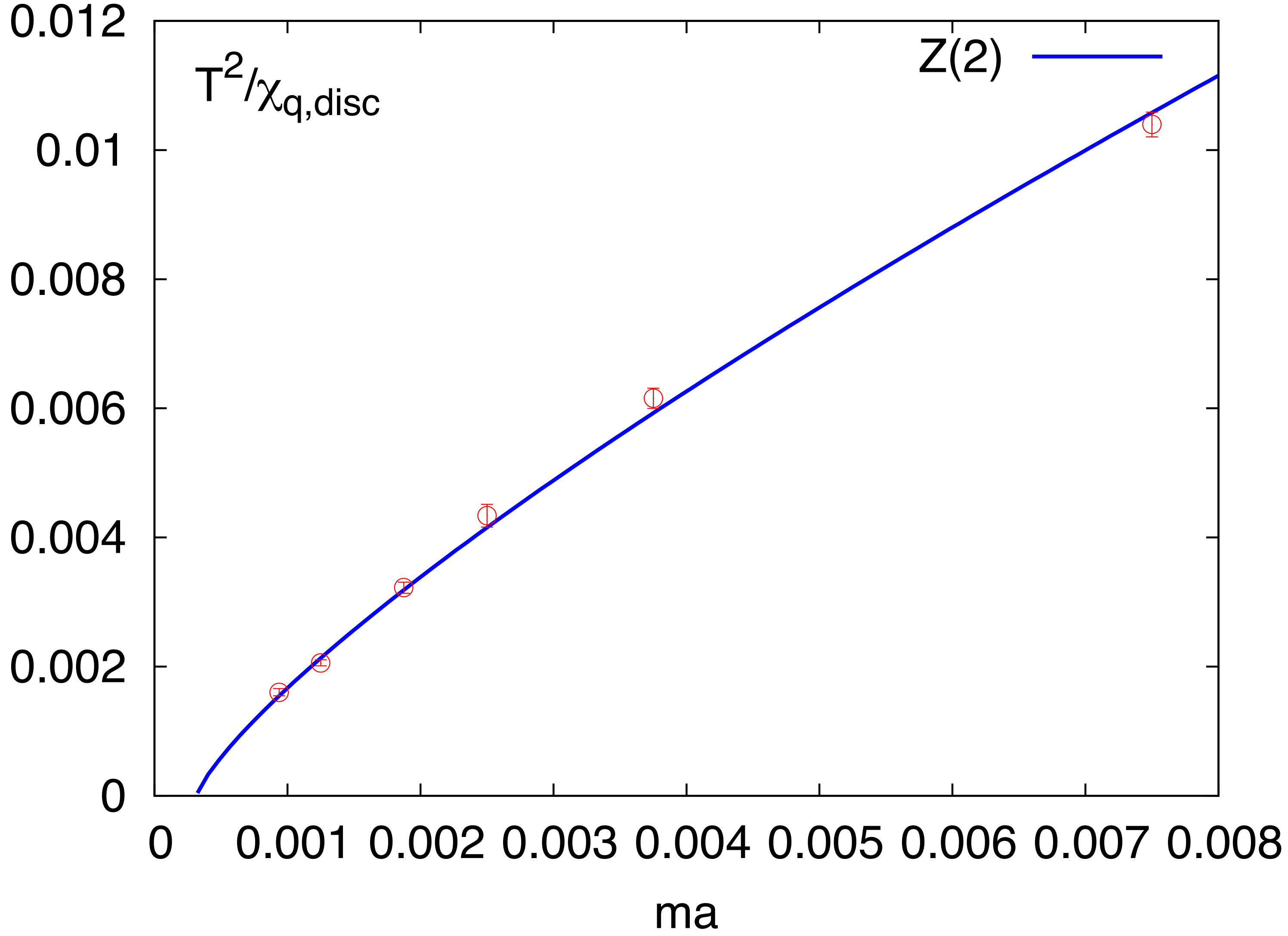
• peak locations in chiral susceptibilities shift to lower temperature with decreasing quark mass

$$\boxed{\chi_q/T^2 \Big|_{T=Tc,mc} \sim (m-m_c)^{1/\delta-1}}$$

• indication of a non-zero critical mass m_c if peak heights in chiral susceptibilities grow faster than $(am)^{1/\delta-1}$:

estimate of the critical mass

Fitting ansatz: $T^2/\chi_q = c\,(m-m_c)^{1-1/\delta}$



Navigation: am=0.0009375 ⇔ m_π =80 MeV

$$\boxed{am_c \approx 0.00037 \qquad m_\pi^c \approx 45\ \text{MeV}}$$

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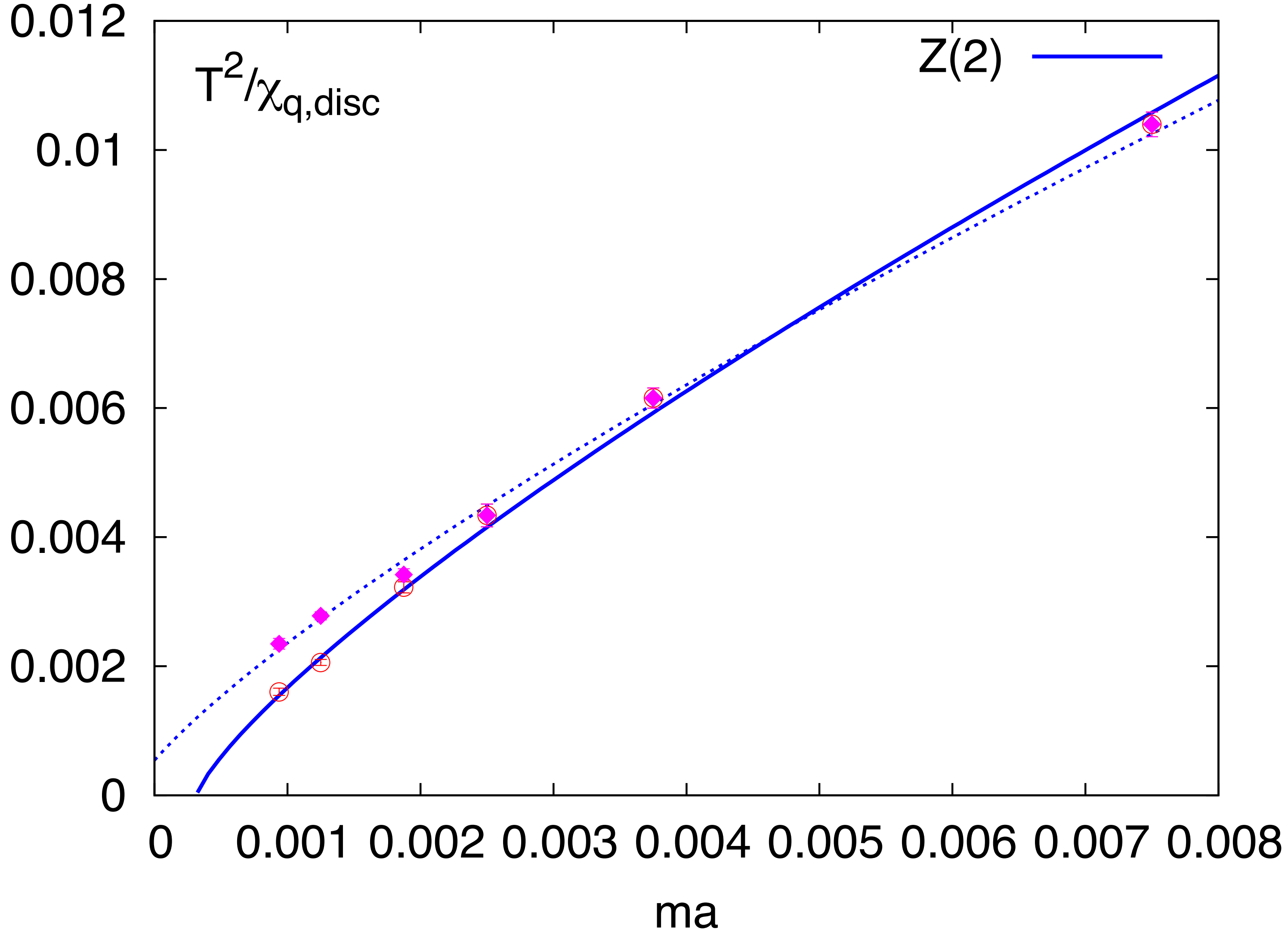


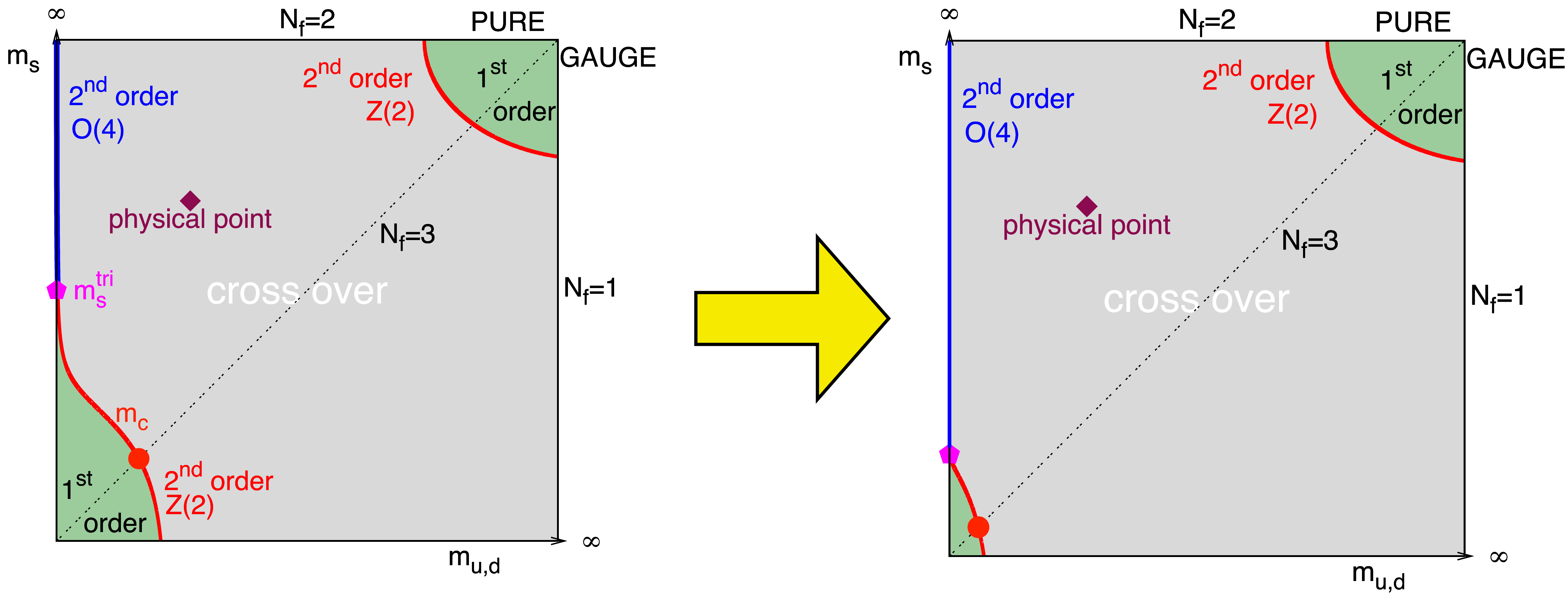
Navigation: am=0.0009375 ⇔ m_π =80 MeV

$am_c \approx 0.00037 \qquad m_\pi^c \approx 45$ MeV

finite V effects

$m_\pi^c \lesssim 45$ MeV

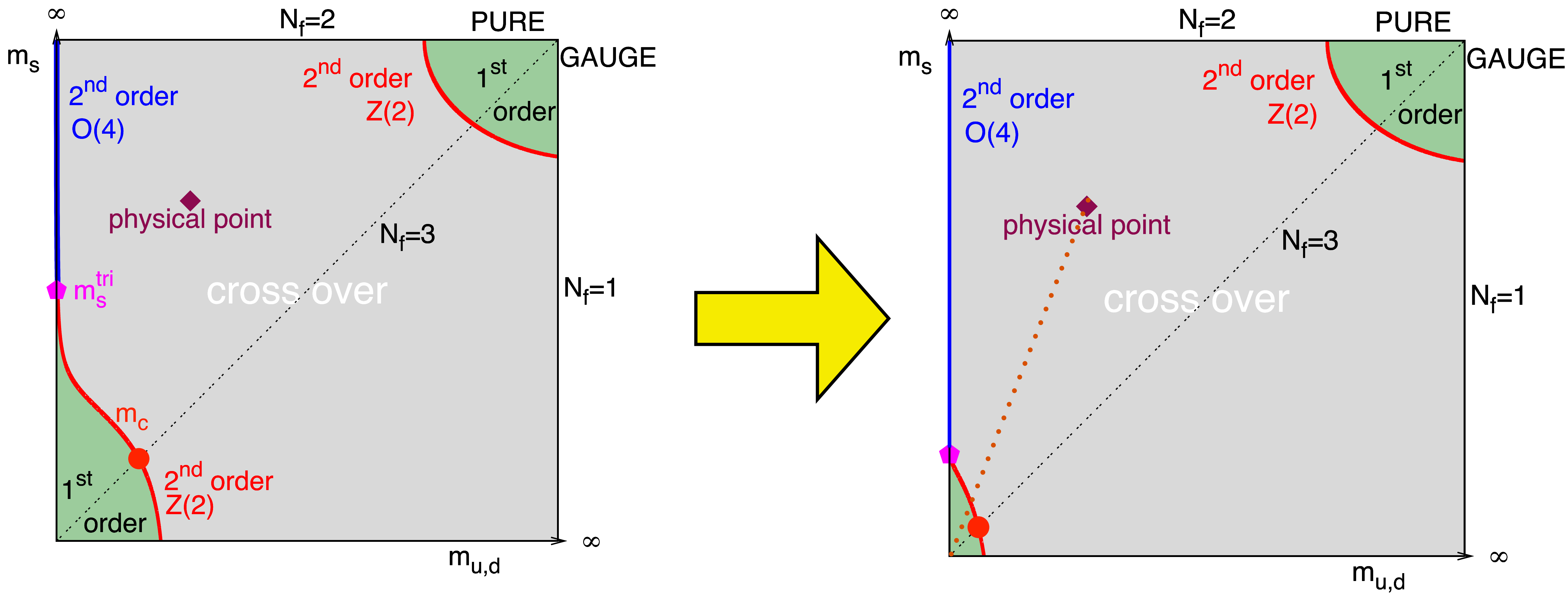
the first order phase transition region revisited



coordinates of the physical point: $(\overline{m}_s/27, \overline{m}_s)$

3 degenerate quarks: coordinates of $m^c_{max} \approx (\overline{m}_s/270, \overline{m}_s/270)$

the first order phase transition region revisited



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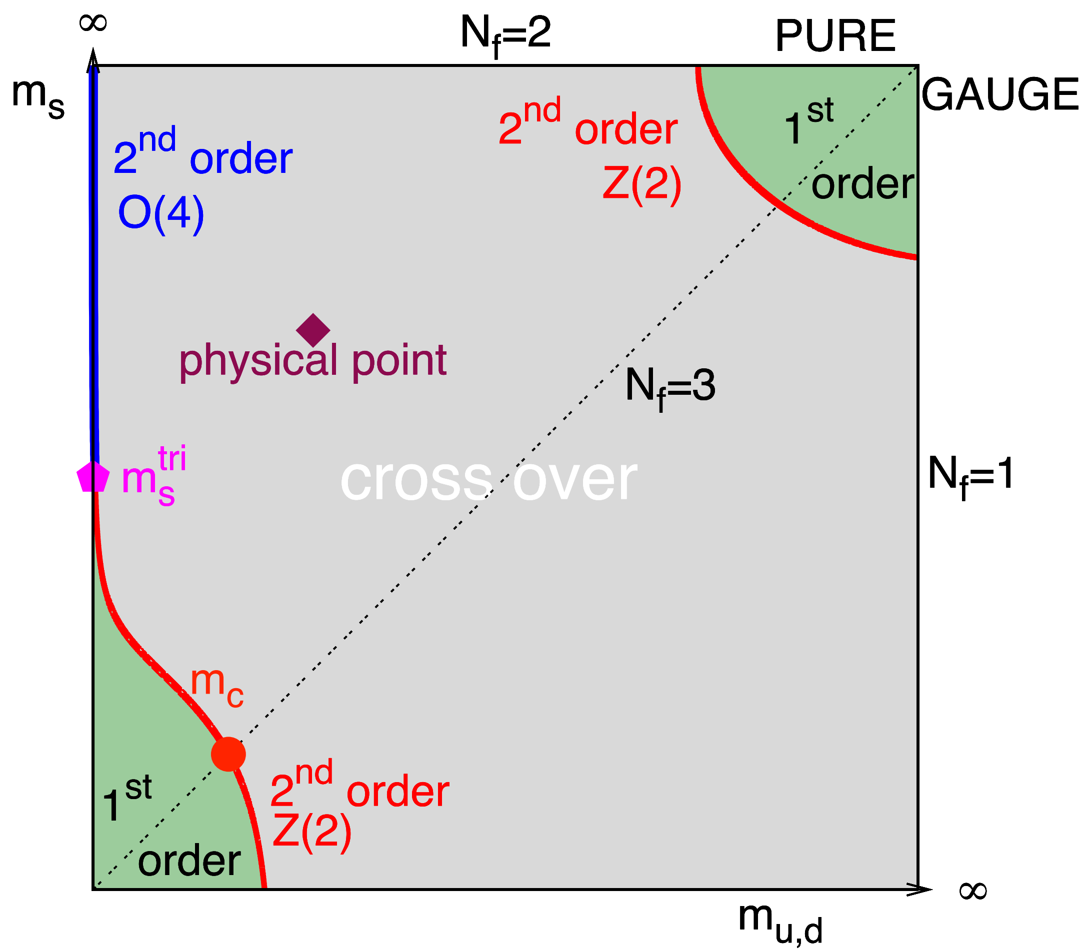
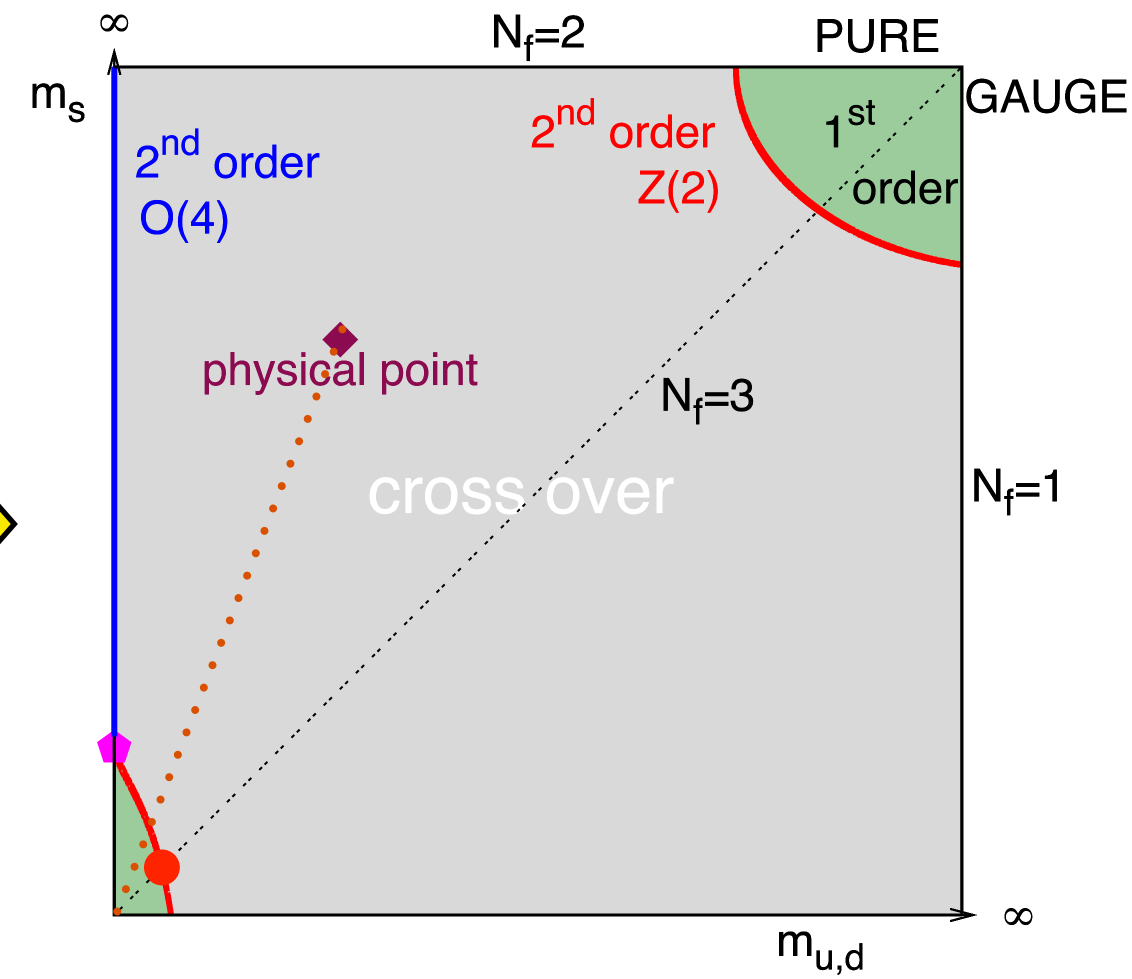
3 degenerate quarks: coordinates of $m^c_{max} \approx (\overline{m}_s/270, \overline{m}_s/270)$

non-degenerate quarks: coordinates of $m^c_{max} \approx (\overline{m}_s/225, \overline{m}_s/8)$ Endrodi et al.,'07

the first order phase transition region revisited





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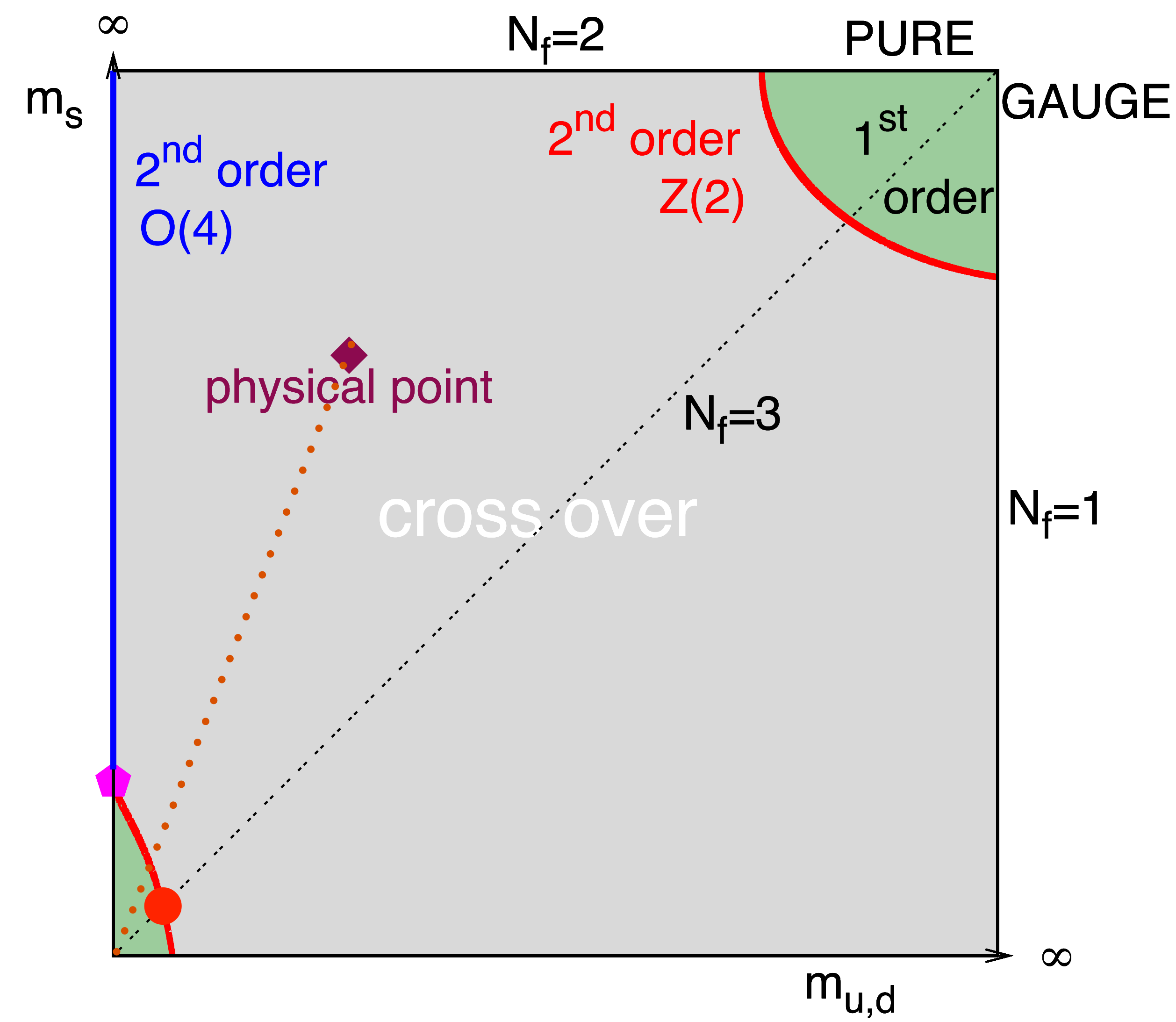
Consequences: to have a critical point at small μ, the critical surface has to bend towards to the physical point with a very large curvature

Summary I

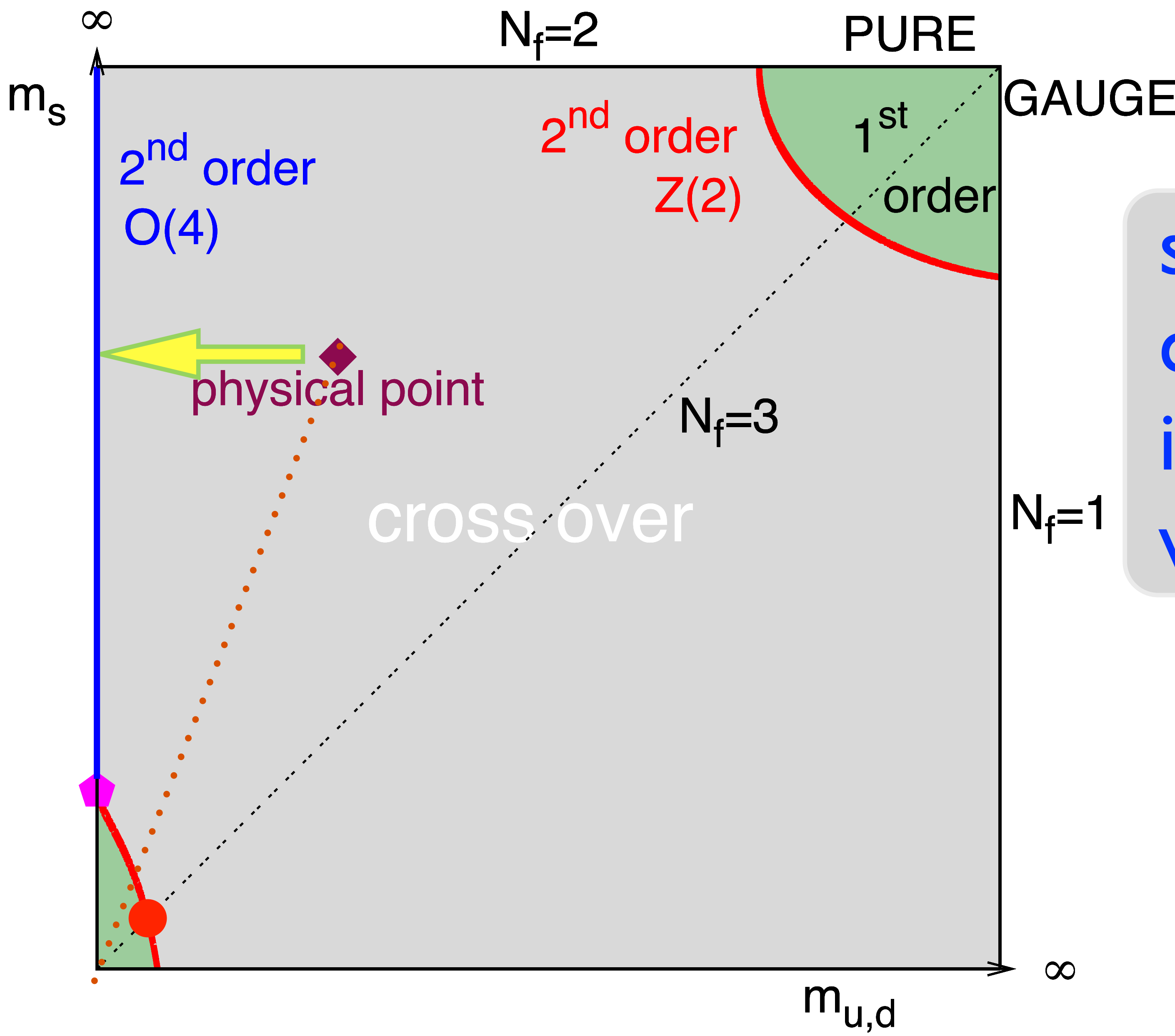
- We study the direct signal for first order phase transition with m_π from 230 MeV down to 80 MeV for N_f=3 on N_τ=6 lattices using the HISQ action

- No evidence for a first order phase transition is found with 230 MeV $\geq m_\pi \geq$ 80 MeV

- From scaling analysis on chiral susceptibility, current estimation gives $m_\pi^c \lesssim$ 45 MeV, which indicates that the first order chiral phase transition region is far away from the physical point



Nf=2+1 QCD



study universal properties of chiral phase transition and its influence to the physical world

- fix the physical strange quark mass
- decrease the light quark mass to the chiral limit

O(N) spin models and Nf=2 QCD

QCD at low energies can be described effectively by O(N) symmetric spin models

- $SU(2)_L \times SU(2)_R$ is isomorphic to O(4)

- O(4) fields: $\sigma=\bar{q}q$, $\pi=\bar{q}\gamma_5 t^i q$

- external field H corresponds to quark mass m

- order parameter "magnetization" $\Sigma=\langle\sigma\rangle$

This description is valid both below and in the vicinity of the chiral phase transition region

chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$$f(m,T)=h^{1+1/\delta}\, f_s(z) + f_{reg}(m,T), \qquad z=t/h^{1/\beta\delta}$$

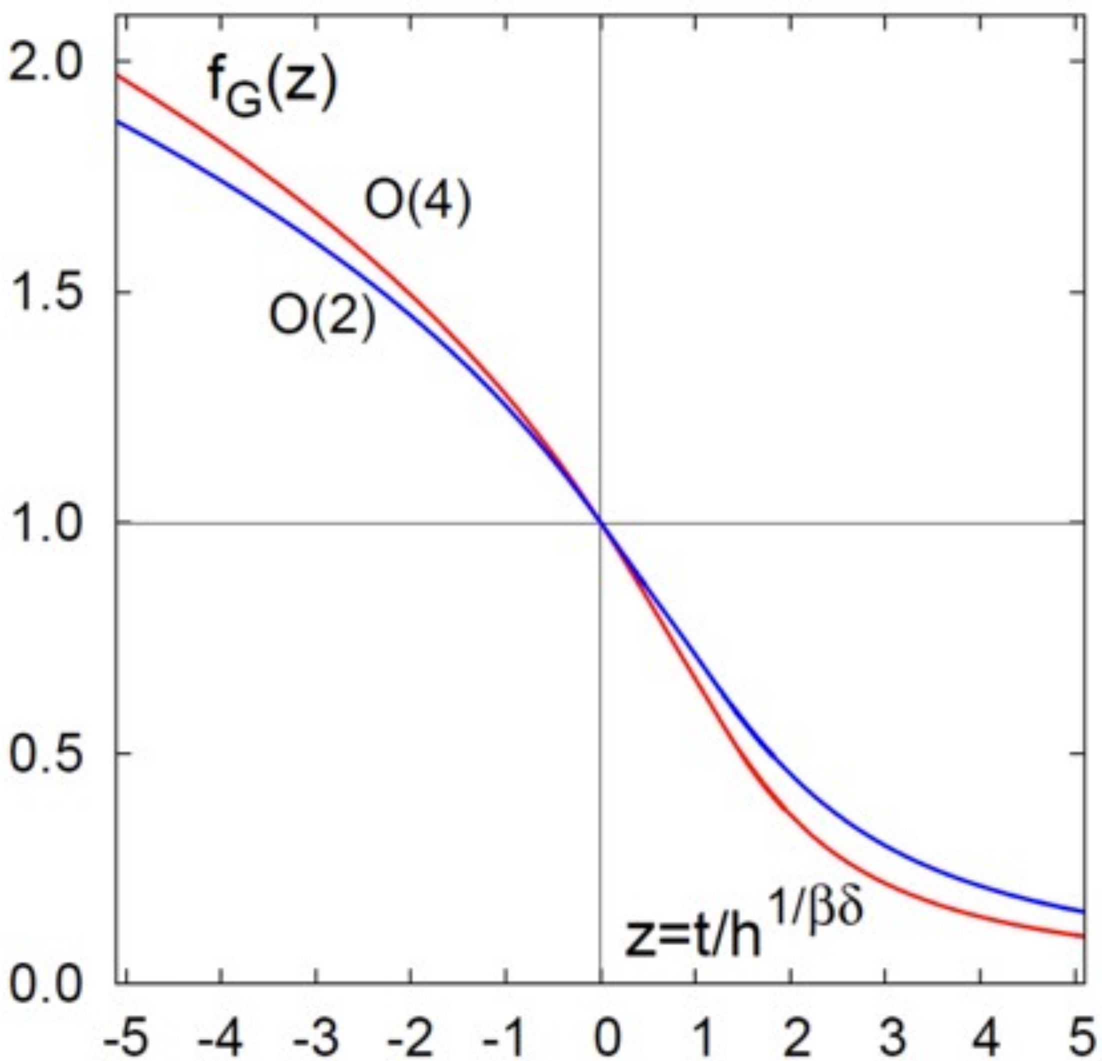
h: external field, t: reduced scaling variable, β,δ: universal critical exponents

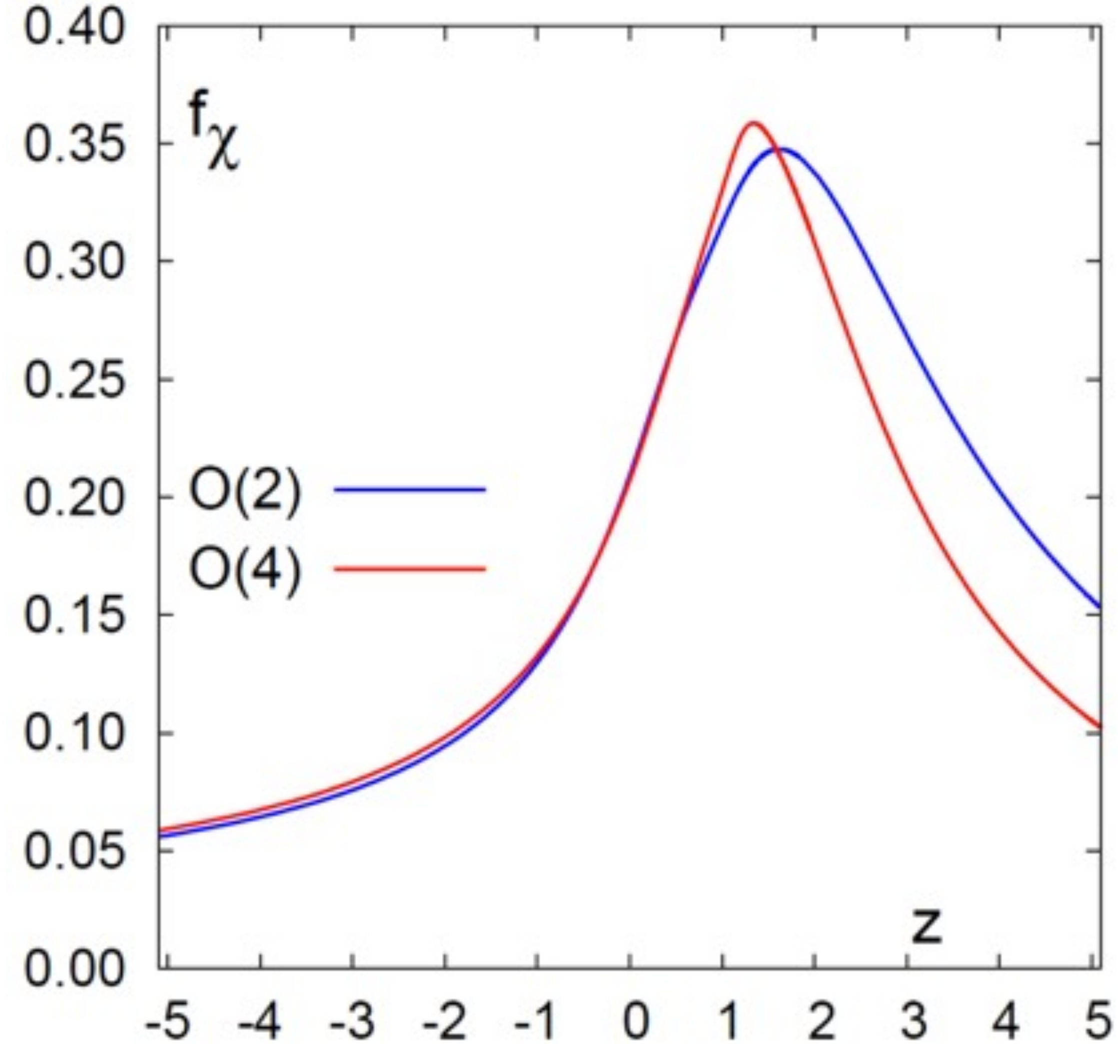
$f_s(z)$: universal scaling function, O(N) etc.

$$h=\frac{1}{h_0}\frac{m_l}{m_s} \qquad t=\frac{1}{t_0}\frac{T-T_c}{T_c}$$

Magnetic Equation of State (MEoS):

$$M = -\partial f_s(t,h)/\partial h = h^{1/\delta}\, f_G(z) \qquad f_\chi(z)=h_0^{1/\delta}(m_l/m_s)^{1-1/\delta}\partial M/\partial h$$





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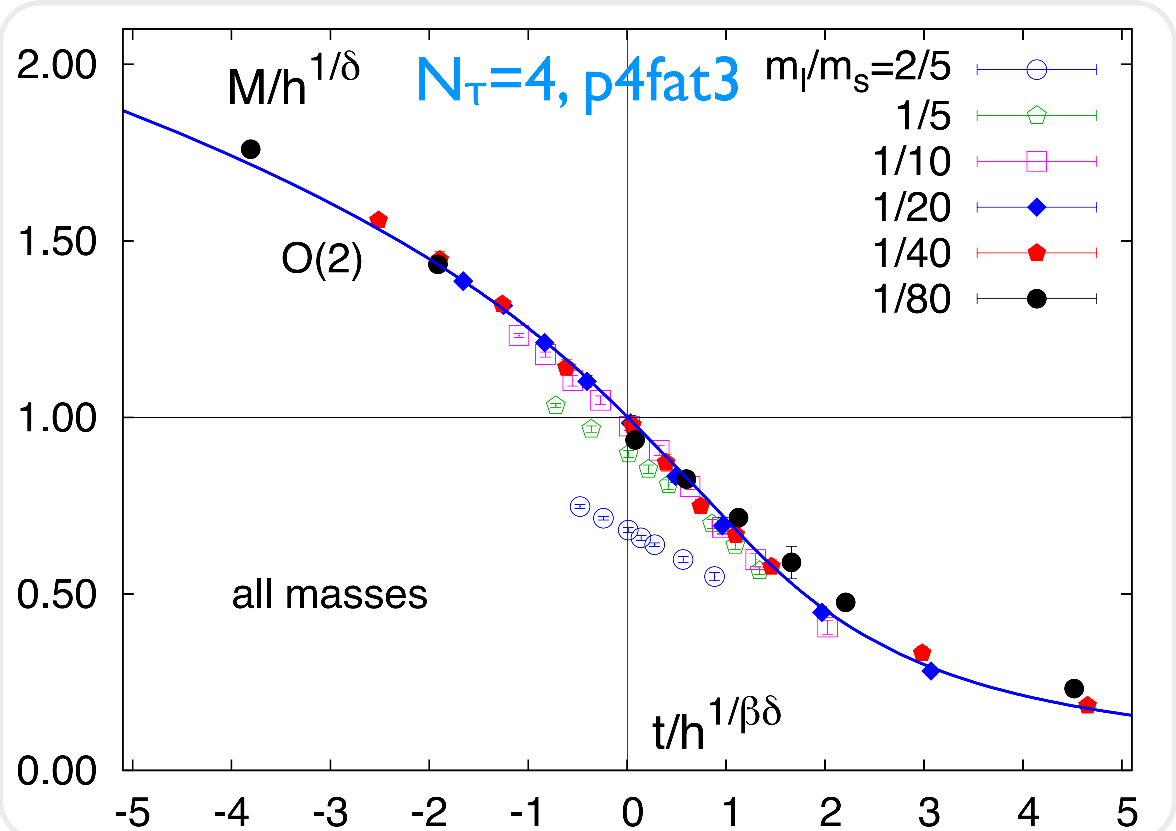
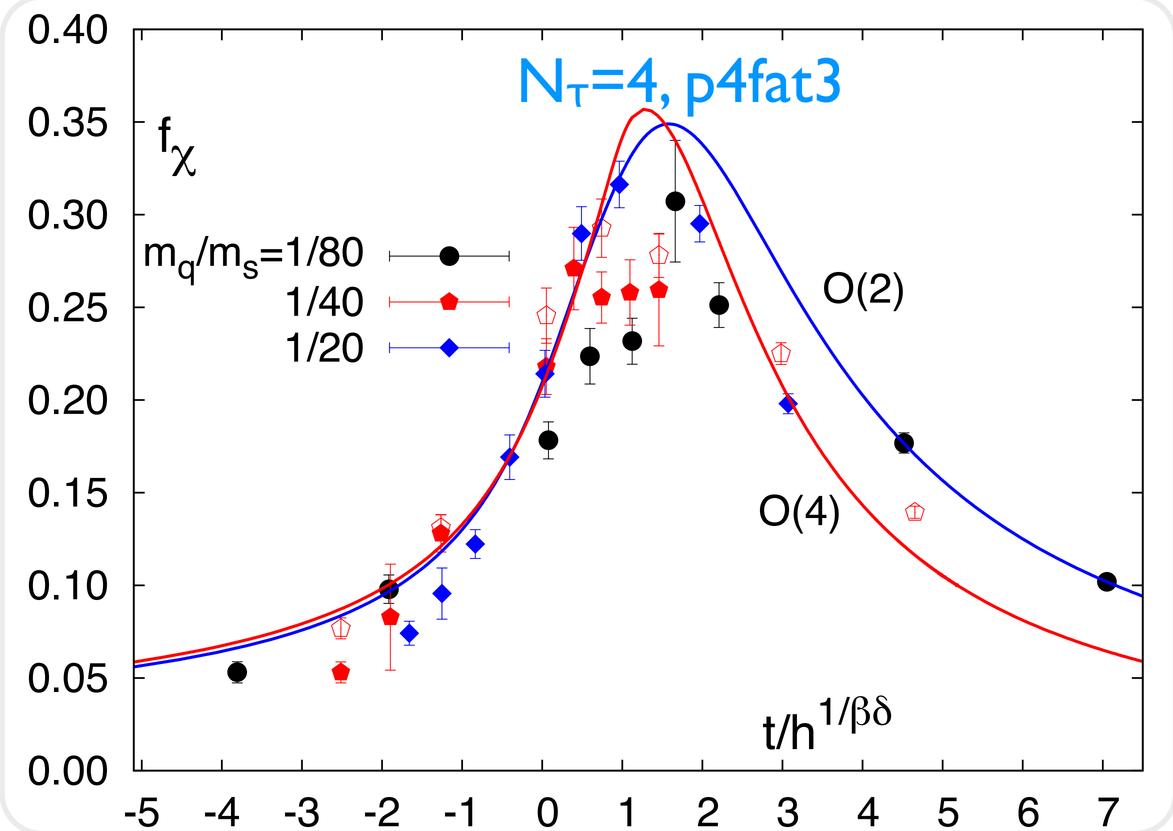


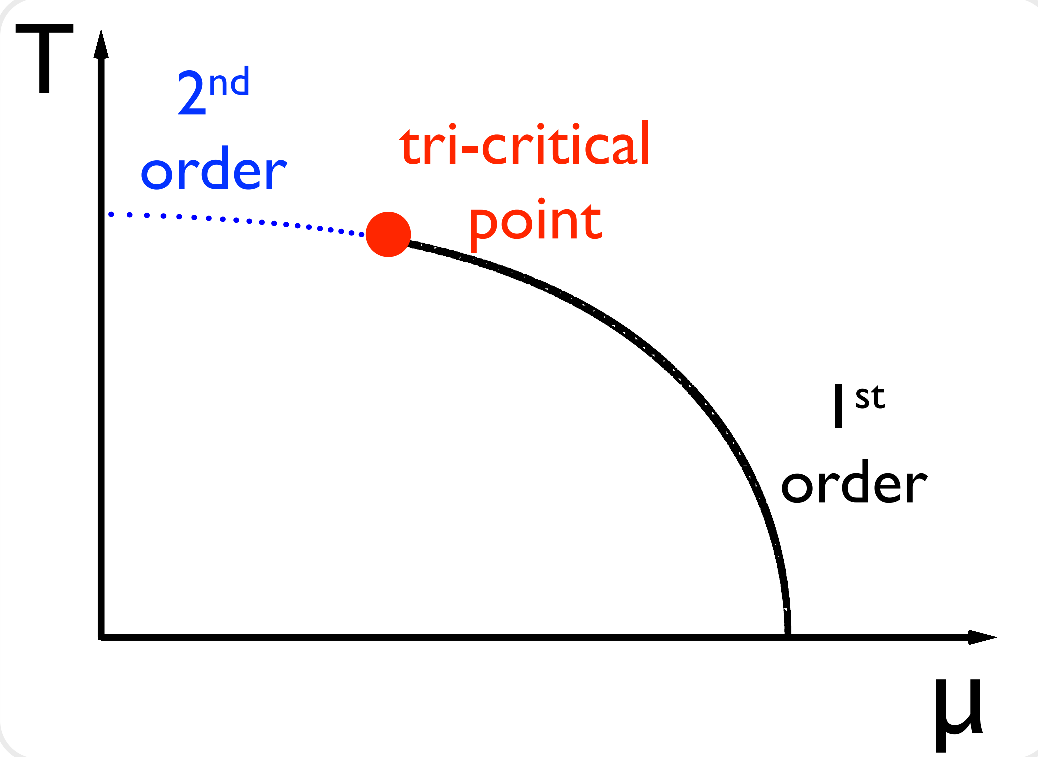


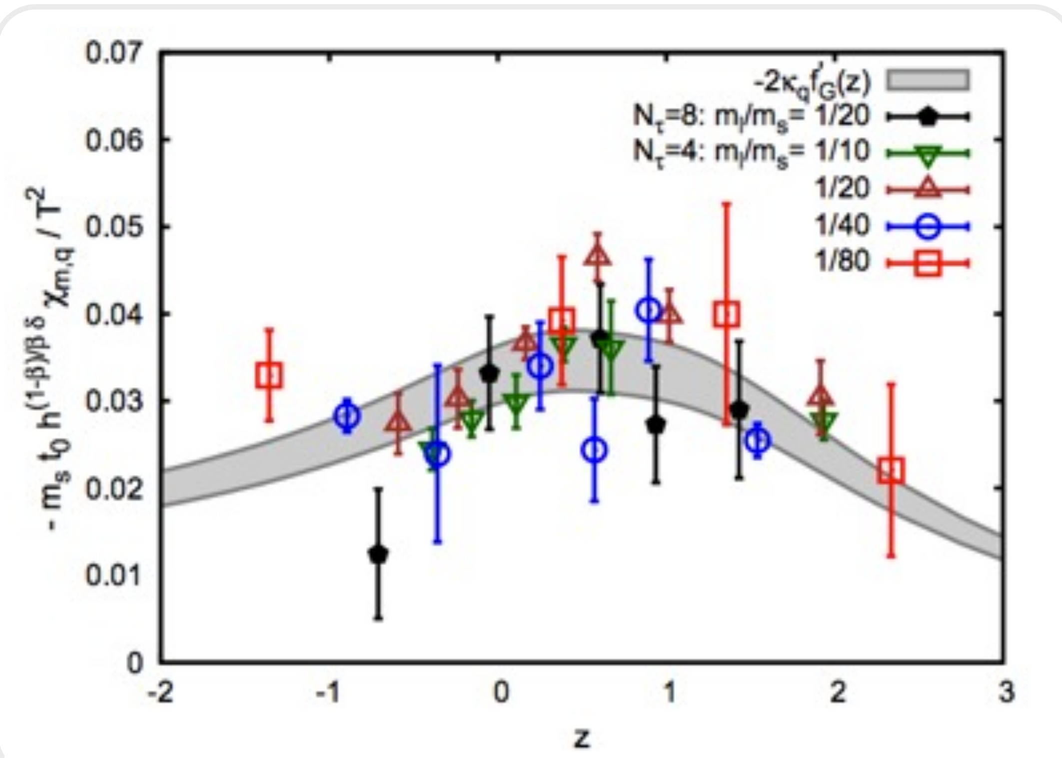
BNL-Bielefeld PRD '09

scaling violations at $m_l/m_s<1/10$

no fitting

universality at small mu





BNL-Bielefeld, O. Kaczmarek et al., PRD 83 (2011) 014504

the curvature of chiral phase transition line: κ_q

$$\frac{T_c(\mu_q)}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^4\right)$$

Taylor expansion of chiral condensate about μ=0

$$\frac{\langle\bar\psi\psi\rangle_l}{T^3} = \left(\frac{\langle\bar\psi\psi\rangle_l}{T^3}\right)_{\mu_q=0} + \frac{\chi_{m,q}}{2T}\left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}((\mu_q/T)^4)$$

Universal scaling

$$\frac{\chi_{m,q}}{T} = \frac{\partial^2\langle\bar\psi\psi\rangle_l/T^3}{\partial(\mu_q/T)^2} = \frac{2\kappa_q T}{t_0 m_s} h^{-(1-\beta)/\beta\delta} f'_G(z)$$

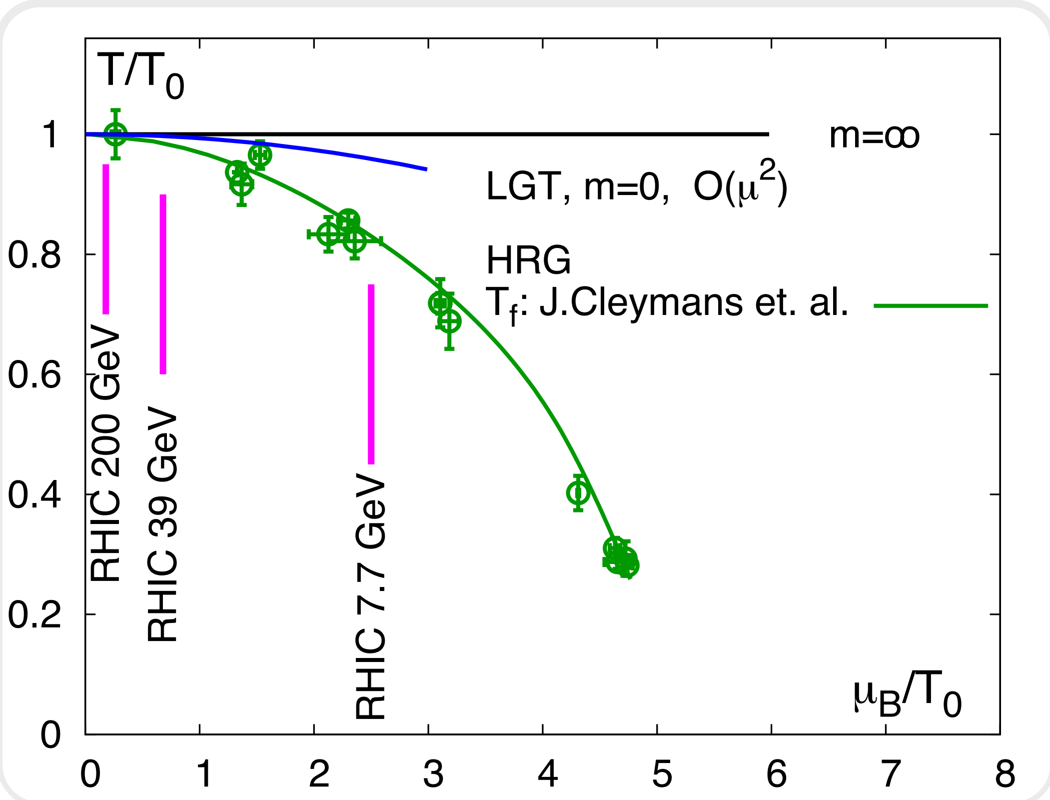
the chiral critical line O. Kaczmarek et al., PRD 83 (2011) 014504

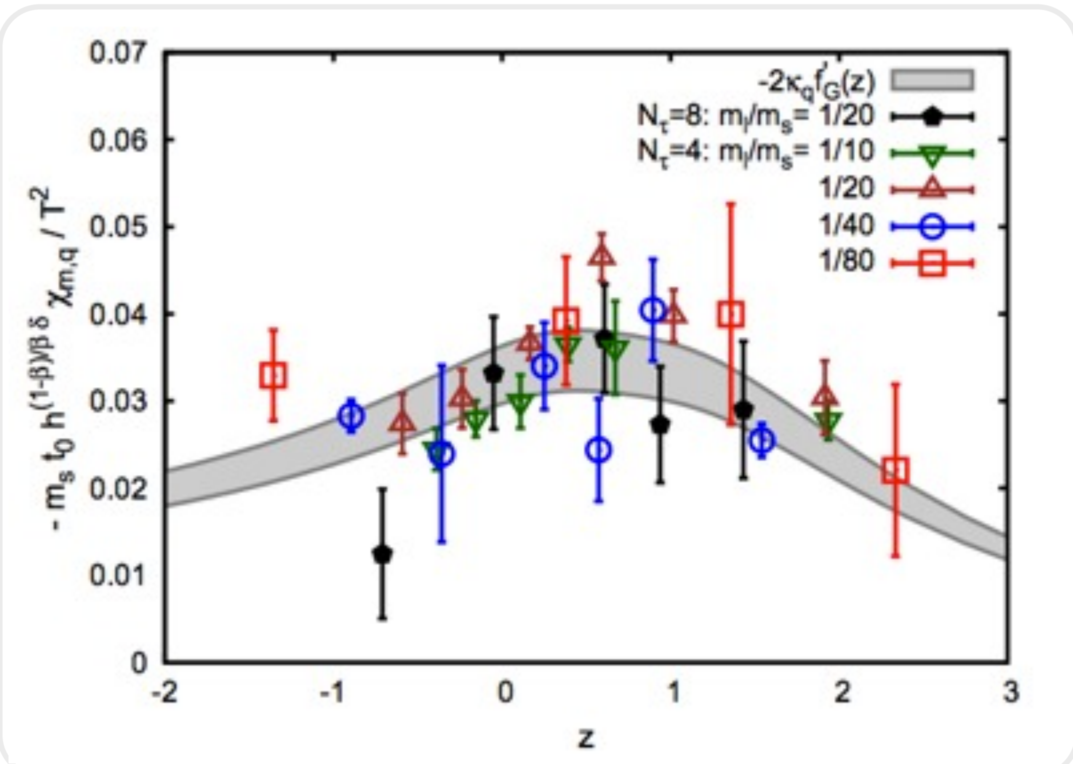
$$\frac{T_c(\mu_q)}{T_c} = 1 - 0.059(2)(4)\left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^4\right)$$

the freeze out curve J. Cleymans et al., PRC 73 (2006) 034905

$$\frac{T_{freeze}(\mu_q)}{T_{freeze}(0)} = 1 - 0.21(2)\left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^4\right)$$

universality at small mu





BNL-Bielefeld, O. Kaczmarek et al., PRD 83 (2011) 014504

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Universal scaling

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data sets for N_f=2+1 on N_τ=6 lattices

- ★ Highly Improved staggered fermions/tree action used
- ★ decrease ml with ms fixed to its physical value, m_π down to 80 MeV
- ★ N_τ=6 lattices

	lattice dim	ms/ml	m_π	#T	# traj.
	24^3x6	20	200 MeV	11	~9000
Volume dep.	12^3x6	27	140 MeV	11	~9000
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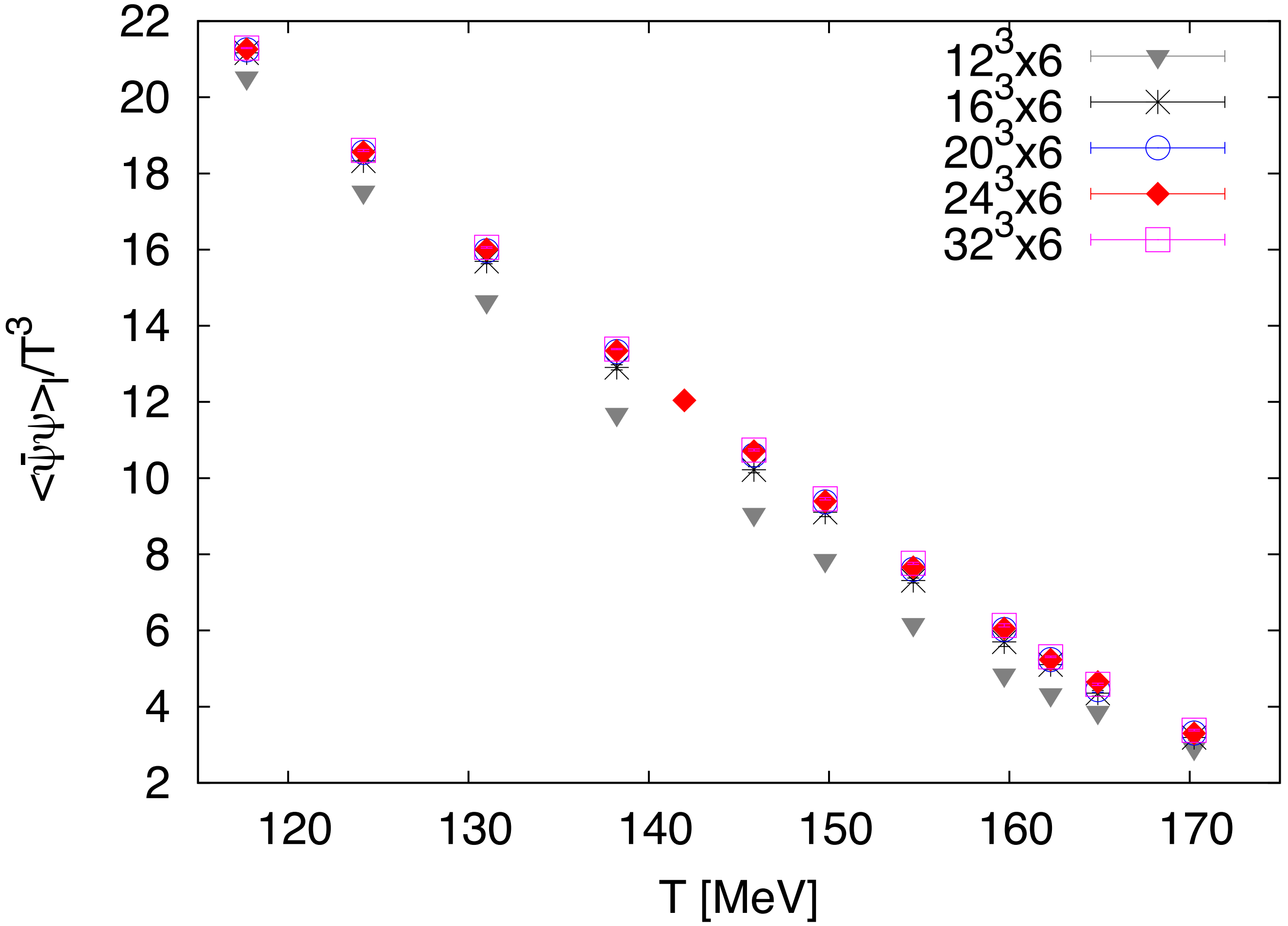
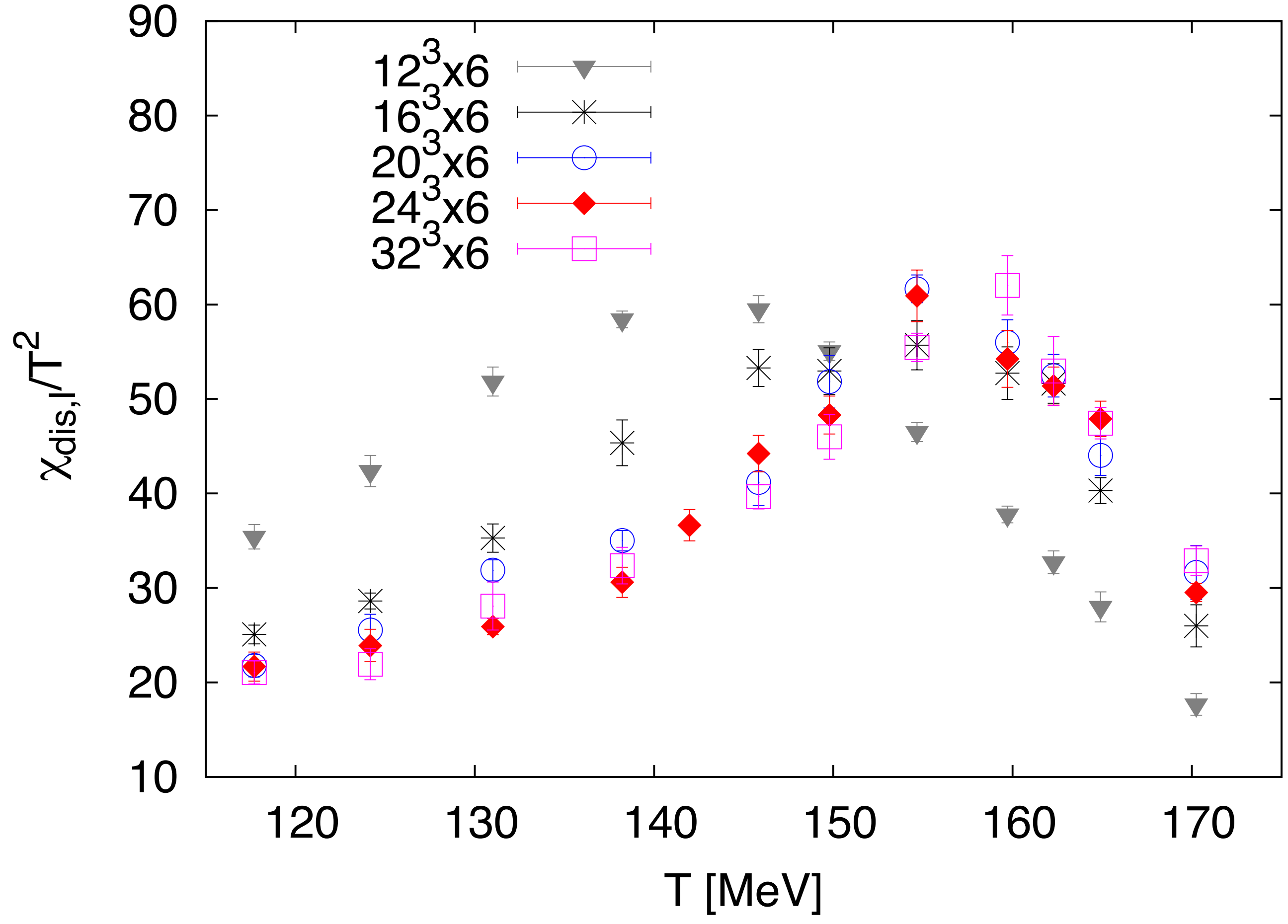
	24^3x6	27	140 MeV	11	~9000
	32^3x6	27	140 MeV	11	~9000
	32^3x6	40	110 MeV	11	~8000
	40^3x6	60	90 MeV	7	~6000
Volume dep.	32^3x6	80	80 MeV	4	~3000
	48^3x6	80	80 MeV	5	~2000

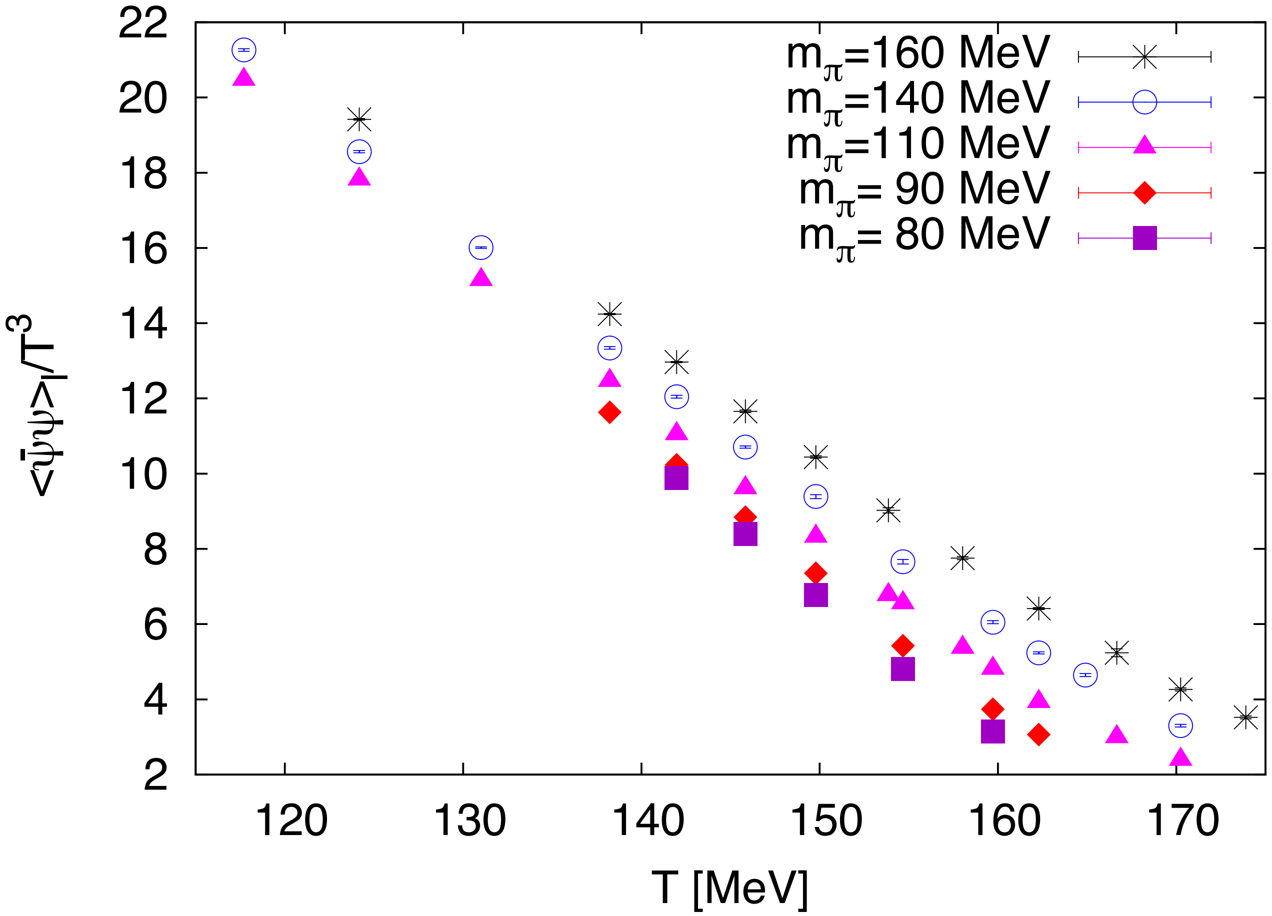
volume dependence at physical pion mass

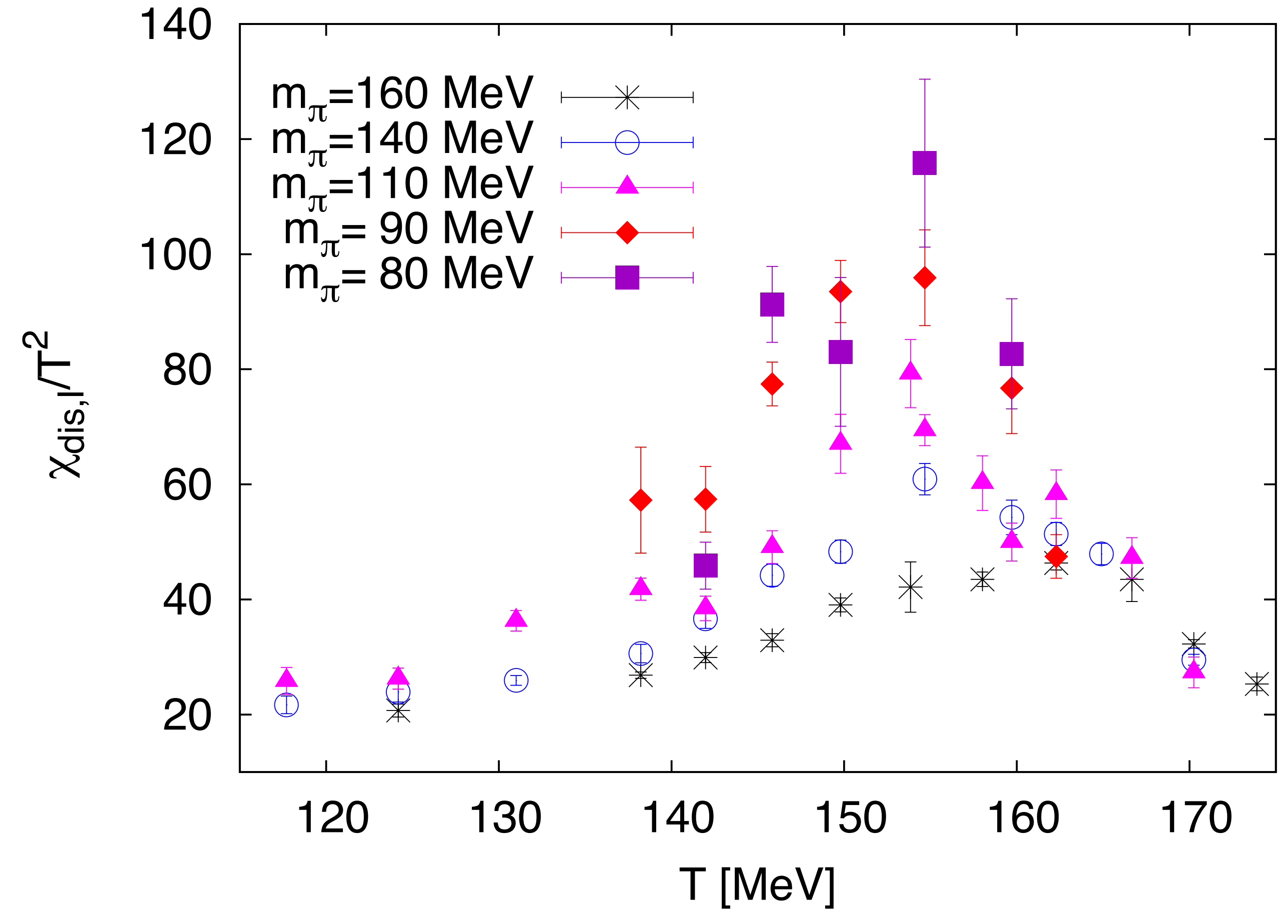




- volume effects are small in 3 largest volume
- m_π L > 4 is ensured in the following datasets
- volume scaling analysis to understand volume effects

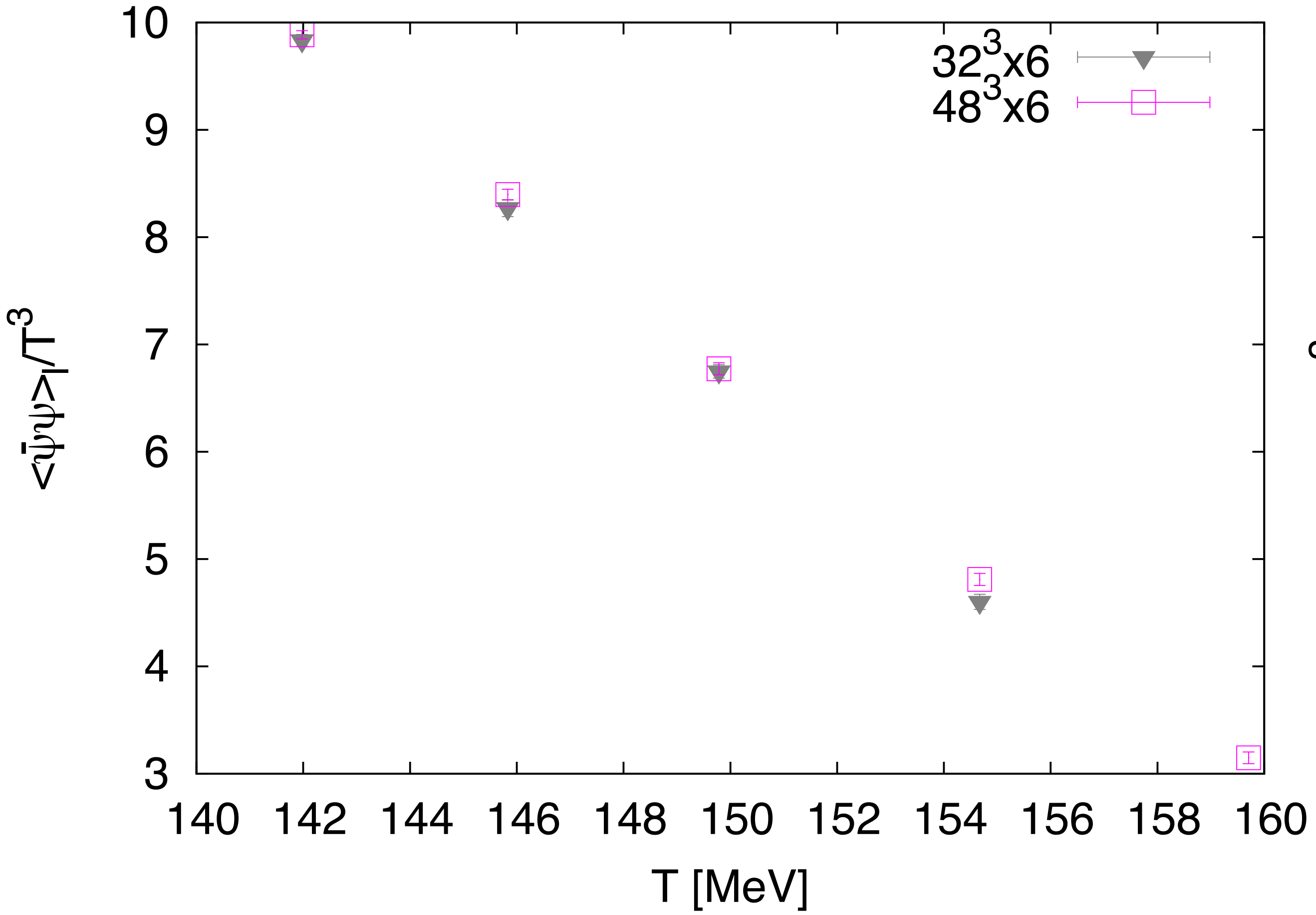
chiral condensates & susceptibilities

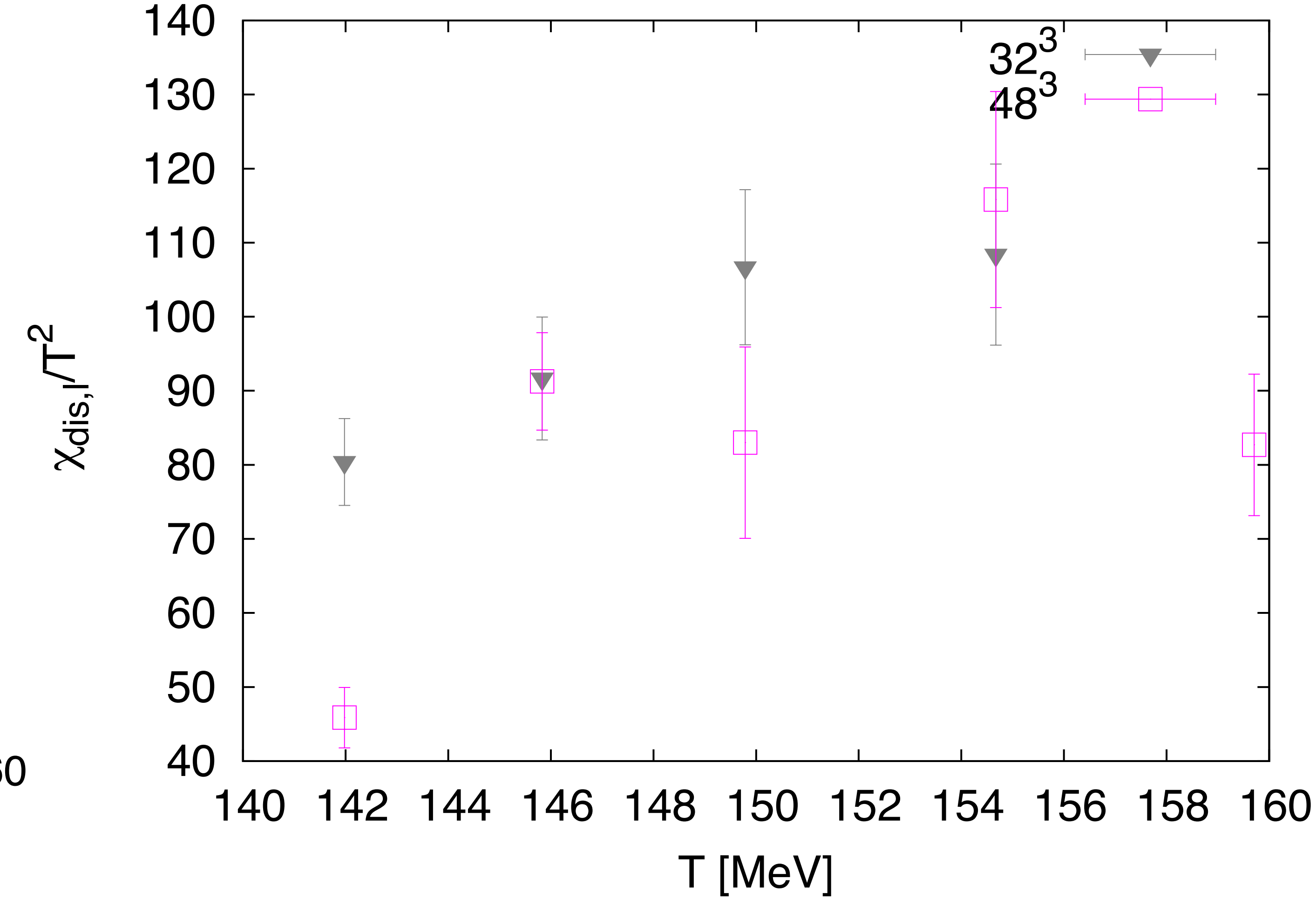




- similar structure as that in Nf=3 case

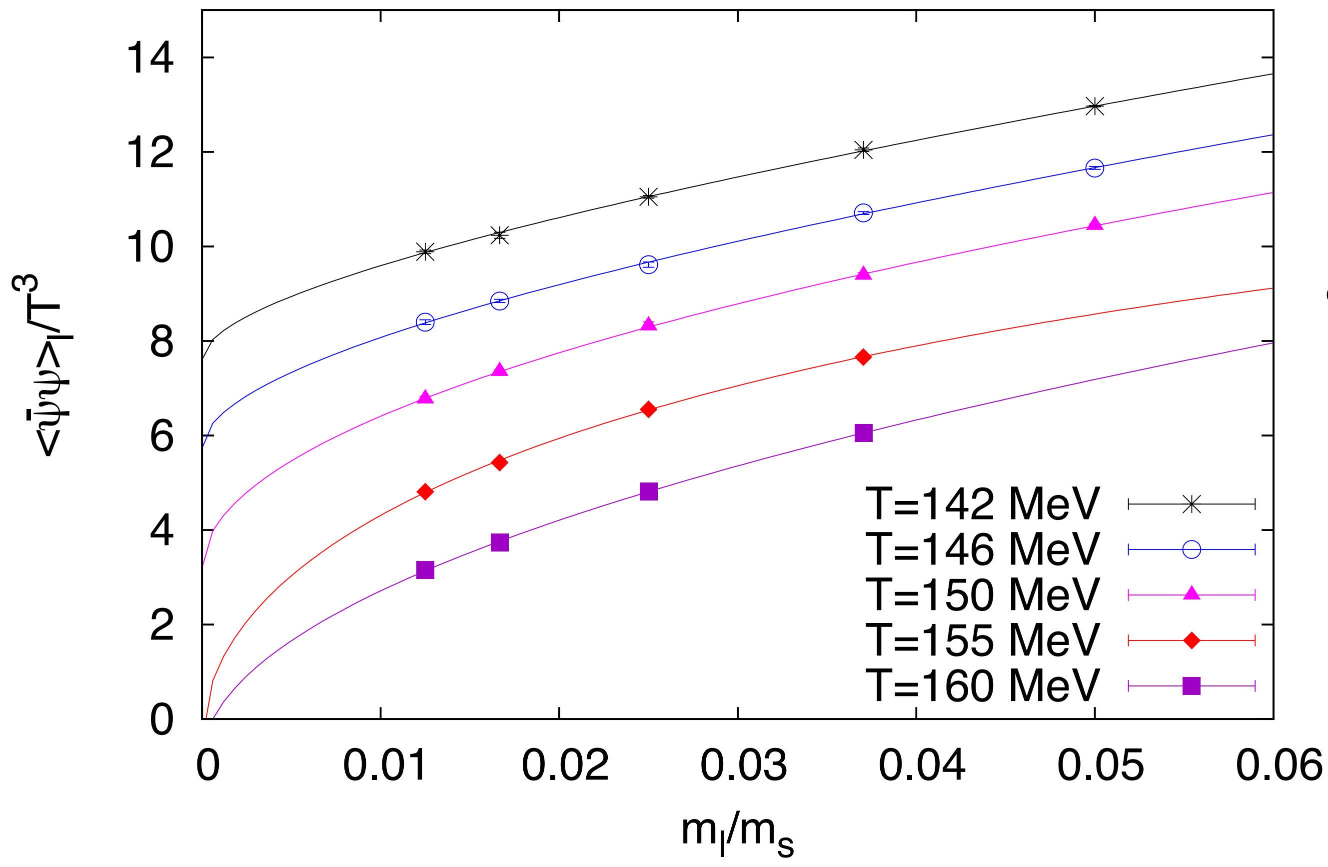
volume dependence at mpi=80 MeV

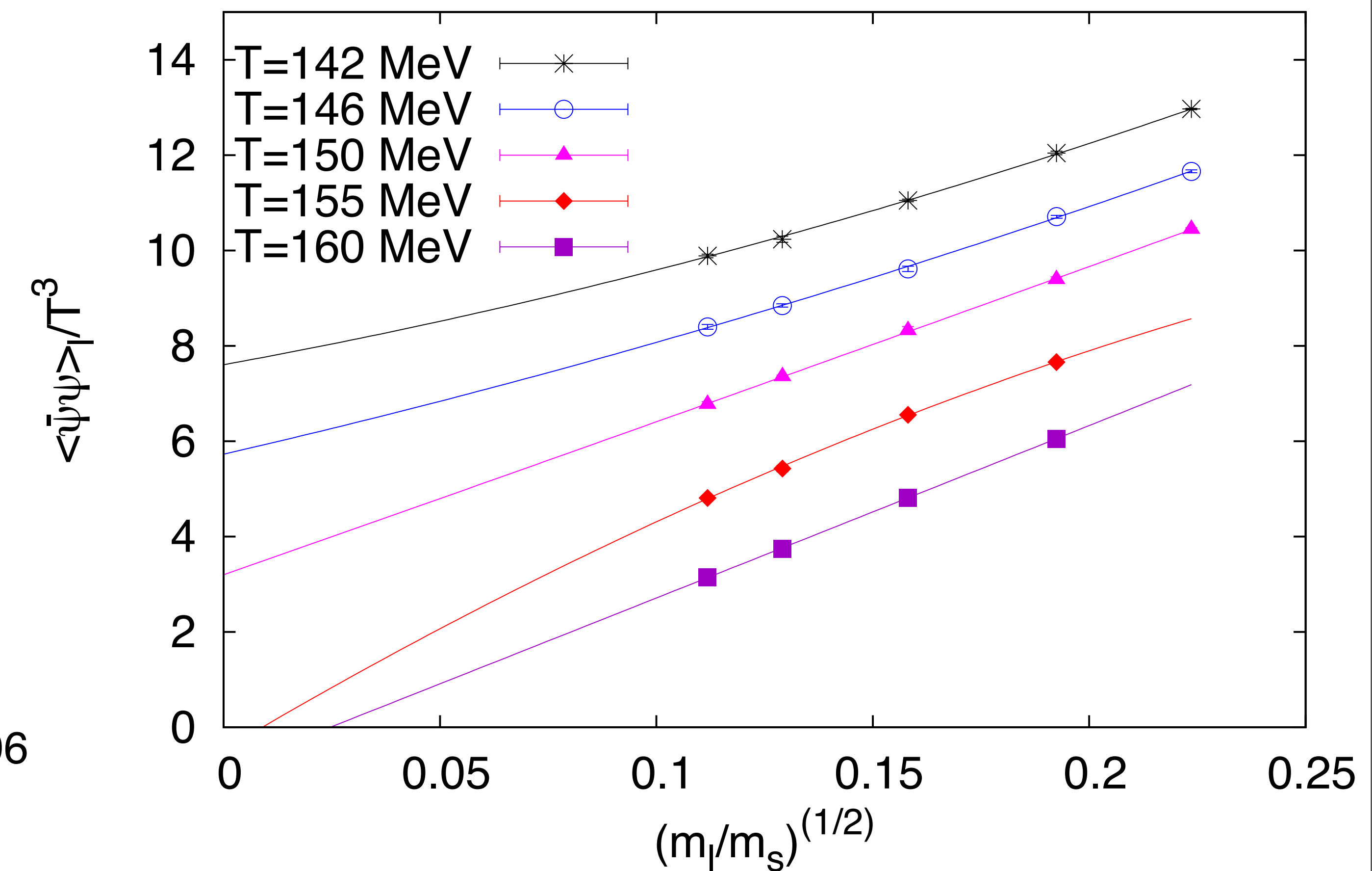




- No first order phase transition at mpi=80 MeV

quark mass dependence of chiral condensates





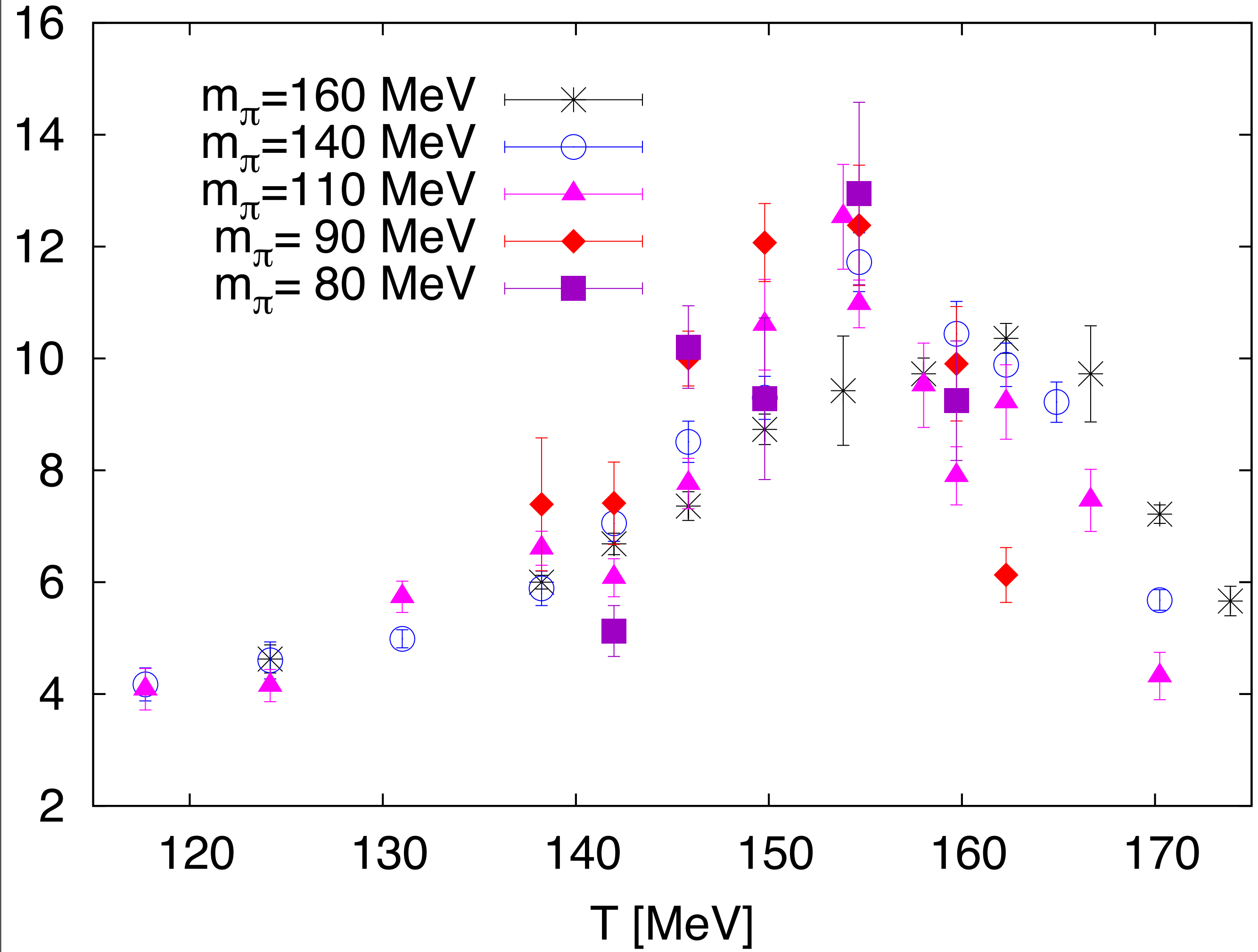
$$\langle\bar{\psi}\psi\rangle_q(T) = \langle\bar{\psi}\psi\rangle_0(T) + c_2(T)m_q + \frac{c_{UV}}{a^2}m_q + \mathcal{O}\left(m_q^3\right) + \delta_{ql}\begin{cases} c_1(T)m_q^{1/2} & T < T_c \\ c_1(T)m_q^{1/\delta} & T = T_c \\ 0 & T > T_c \end{cases}$$

fit ansatz:

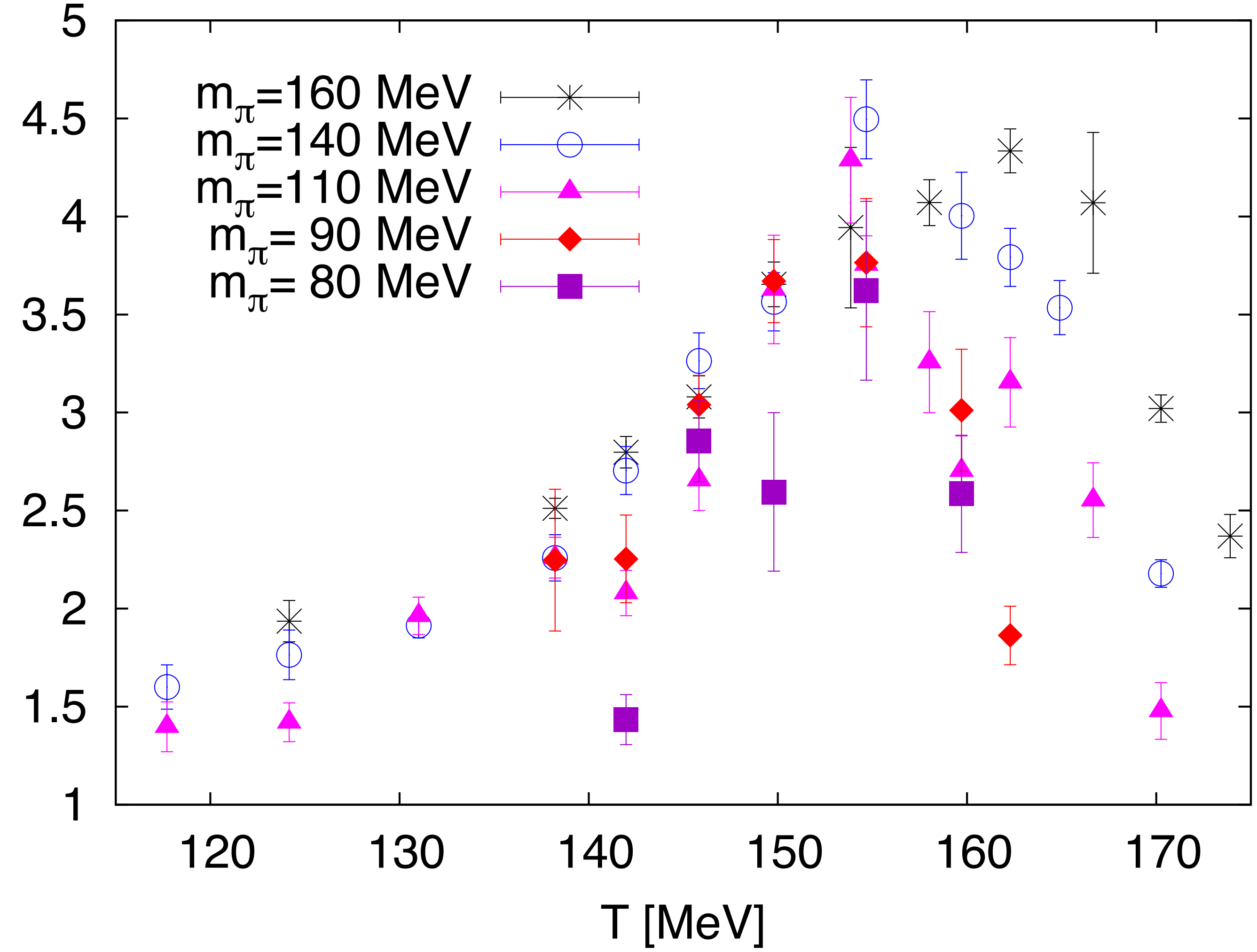
$$a + b\left(\frac{m_l}{m_s}\right)^{1/2} + c\,\frac{m_l}{m_s}$$

rescaled chiral susceptibilities

$\boxed{\chi_{l,disc}(m_l/m_s)^{1/2}}$

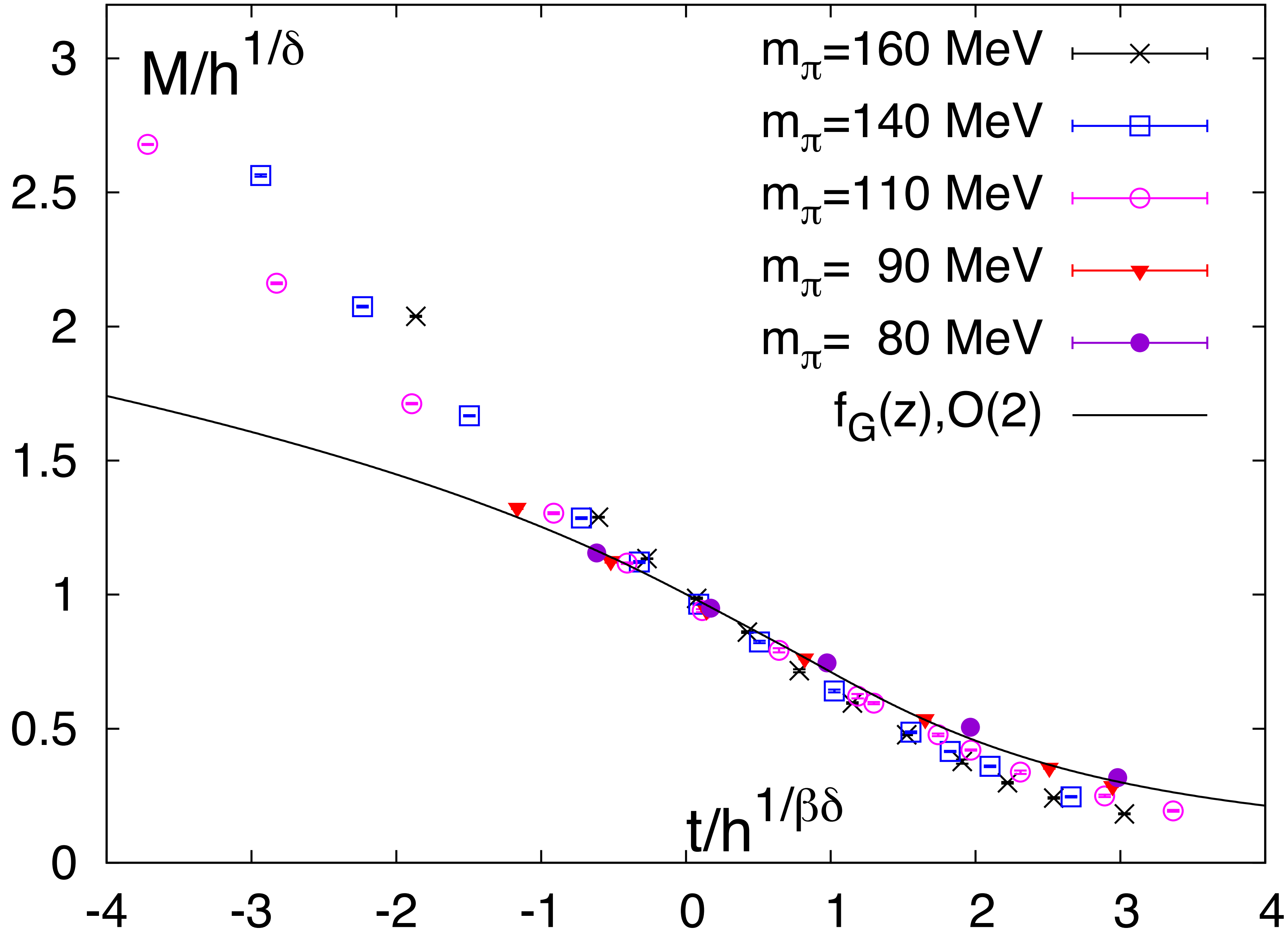


$\boxed{\chi_{l,disc}(m_l/m_s)^{-1/\delta+1}}$



$$\chi_q(T) = c_2(T) + \frac{c_{UV}}{a^2} + \mathcal{O}\left(m_q^2\right) + \delta_{ql} \begin{cases} \frac{c_1(T)}{2} m_q^{-1/2} & T < T_c \\ \frac{c_1(T)}{\delta} m_q^{1/\delta-1} & T = T_c \\ 0 & T > T_c \end{cases}$$

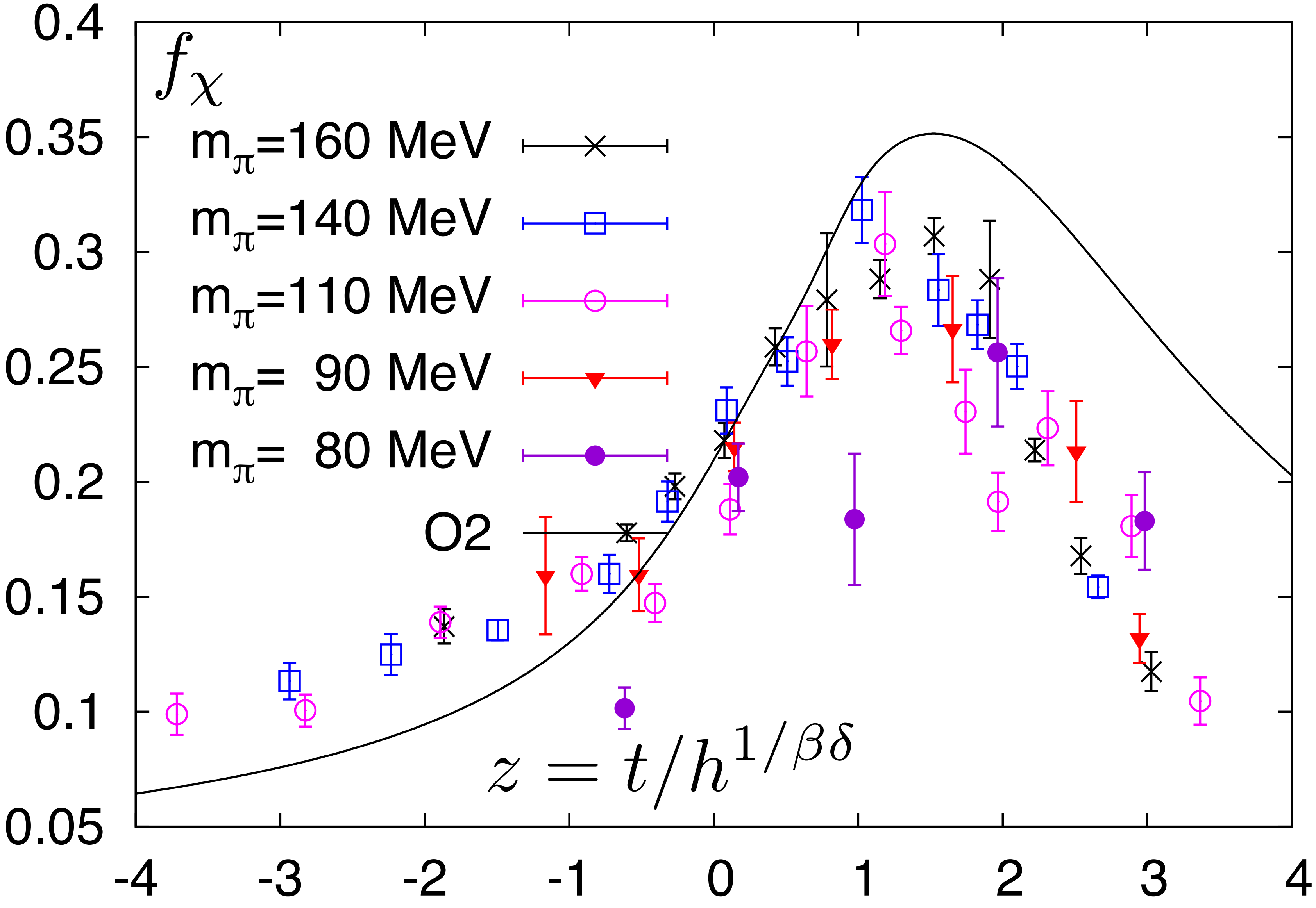
O(2) scaling analysis



parameters t0, h0, Tc obtained from the scaling fits of the chiral condensates for our lightest two pion masses: 90 and 80 MeV

system with mpi=160, 140, 110 MeV does not lie in the scaling window

O(2) scaling analysis



parameters t0, h0, Tc obtained from the scaling fits of the chiral condensates for our lightest two pion masses: 90 and 80 MeV

system with mpi=160, 140, 110 MeV does not lie in the scaling window

Summary II

- We study the chiral observables on Nt=6 lattices with mpi=160,140,110, 90 and 80 MeV

- No direct signal of a first order phase transition in current pion mass window is found

- The system with mpi=160, 140, 110 MeV seems not lie in the scaling regime

- A detailed study on the scaling violation is needed to account the influence of chiral phase transition to the physical world

- To study the universal properties of chiral phase transition simulations with pion masses lower than 80 MeV are crucially needed